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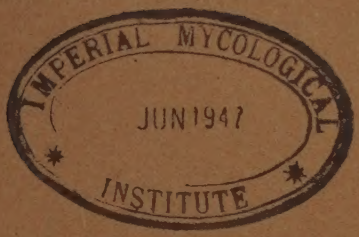

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MATHEMATICS

ON SOME TOPOLOGIC INVARIANTS OF RIEMANNIAN MANIFOLDS

By L. PONTRJAGIN, Corresponding Member of the Academy

The results stated in the present note are essentially a translation of my results of earlier date (1) into the language of differential forms. In substance this note is independent of the preceding one.

Let M be a differentiable closed orientated manifold, and $V = \{V_{i_1, i_2, \ldots, i_r}\}$ a skew-symmetric covariant tensor field of order r defined on it (in the sequel V shall be simply called a field). We shall say that V is closed, if its outer (alternated) derivative is equal to zero. A closed field V shall be called homologous to zero, if it is equal to the outer derivative of another field. Two closed fields shall be called homologous to each other, if their difference is homologous to zero. The classes of closed fields of order r homologous to each other form a linear space of dimensionality p_r, wnere p_r is the r-dimensional Betti number of the manifold M. This fact has been established by de Rham (2), in whose paper will be found a detailed exposition of all results pertaining to this problem.

Instead of the skew-symmetric tensor V it is often more convenient to consider the corresponding skew-symmetric multilinear form in the vectors $x_1, x_2, \ldots, x_r$:

$$F(x_1, x_2, \ldots, x_r) = V_{\alpha_1, \alpha_2, \ldots, \alpha_r} x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_r^{\alpha_r}$$

If M is a homogeneous manifold, *i. e.* a compact transitive Lie group G of transformations of M being given, there naturally arises the notion of the field invariant with respect to the group G. Cartan (3) has proved the following fundamental theorem:

For every closed field there exists an invariant closed field homologous to it. If the invariant closed field is homologous to zero, then it is the outer derivative of an invariant field.

This theorem gives us the possibility, in considering the homologies in a homogeneous manifold M, to confine ourselves to invariant fields. Since an invariant field is determined by its value at one point $p \in M$, the whole problem is reduced to the consideration of constant tensors (independent of the point of the manifold M) defined at this point p.

Let $H(k, l)$ be the manifold of all orientated k-dimensional planes drawn through the fixed point o of the Euclidean space R^{k+l} of $k+l$ dimensions. If we take G to be the group of all rotations of the space R^{k+l} about the point o, then $H(k, l)$ becomes a homogeneous space. Choose in R^{k+l} a fixed orthogonal system of coordinates and denote by $e_1, e_2, \ldots, e_k$ the first k unit vectors of this system; let further R_0^k, $R_0^k \in H(k, l)$ be the k-dimensional orientated plane drawn through the end-points of these vectors. We take in the sequel the element R_0^k of the manifold $H(k, l)$ for the fundamental point $p \in H(k, l)$. We now introduce coordinates in the neighbourhood of the

element R_0^k of the manifold $H(k, l)$. Let R_ξ^k be an arbitrary element from $H(k, l)$; whose orthogonal projection on R_0^k is positive. By e_i' we denote the vector from R_ξ^k, whose orthogonal projection on R_0^k coincides with e_i'. The vectors $e_1', e_2', \ldots, e_k'$ are determined by the plane R_ξ^k and themselves determine it. The $(k+j)$-th component of the vector e_i' we denote by $x_{i,j}$. The elements of the matrix $||x_{ij}||$ $(i=1, 2, \ldots, k;\ j=1,2, \ldots, l)$ are the coordinates of the plane R_ξ^k.

The coordinates of the vector x given at the point R_0^k of the manifold $H(k,l)$ we shall also take to be the elements of the matrix $\|x_{ij}\|$.

Theorem 1. *Let $l \geqslant k+2$; then all skew-symmetric multilinear forms of degree not exceeding k in vectors at the point R_0^k of the manifold $H(k, l)$, generating closed invariant fields in $H(k, l)$, may be defined in the following manner.*

Let $x^p = \| x_{ij}^p \|$ $(p=1, 2, \ldots, k)$ be a system of k variable vectors defined at the point R_0^k. Put

$$x_{\alpha\beta}^{pq} = \sum_{j=1}^{l} x_{\alpha j}^p x_{\beta j}^q$$

If k is even, form the expression $x_{1,2}^{1,2} x_{3,4}^{3,4} \ldots x_{k-1,k}^{k-1,k}$; the skew-symmetric form obtained from this expression by alternation over all vectors x^p we denote by K. Let now $r=2s \leqslant k$ be a natural number divisible by four. Form the expression

$$\sum_{(\alpha_1 \ldots \alpha_s)=1}^{k} x_{\alpha_1,\alpha_2}^{1,2} x_{\alpha_2,\alpha_3}^{3,4} x_{\alpha_3,\alpha_4}^{5,6} \ldots x_{\alpha_s,\alpha_1}^{r-1,r}$$

The skew-symmetric form obtained from this expression by alternation over all vectors x^p we denote by L^r. If L^r and L^s are two forms of the indicated type, and $r+s \leqslant k$, then choosing from them different arguments we may form their product and alternate it. The skew-symmetric form thus obtained we denote by $L^{r,s}$. In the same way we may construct the form $L^{r_1, r_2, \ldots, r_t}$, if $r_1+r_2+\ldots+r_t \leqslant k$.

Thus, we have skew-symmetric forms of the kind

$$K;\ L^{r_1, r_2, \ldots, r_t} \tag{1}$$

Herein K exists only for even k, and the numbers $r_1, r_2, \ldots, r_t$ are all divisible by four, and $r_1+r_2+\ldots+r_t \leqslant k$.

The forms of the kind (1) give in their aggregate a linearly independent basis of the set of all forms in question. Moreover, they are linearly independent in the sense of homologies.

Proof. Denote by G_0 the subgroup of the group G composed of all rotations transferring R_0^k into itself. On the theory of invariants (4) it may be easily established that every skew-symmetric form in the vectors $x^1, x^2, \ldots, x^k$ of the dimensionality less than or equal to k invariant with respect to the group G_0, is linearly expressible through forms of the kind (1). Since the forms are all of even degree, the invariant fields generated by them are closed.

The homologous independence of the forms (1) may be established on the ground that the number of these forms of degree r is equal to the Betti number of dimensionality r of the manifold $H(k, l)$(1).

Theorem 2. *Let M^k be an orientated k-dimensional manifold situated in the Euclidean space R^{k+l} of dimensionality $k+l$. We denote the Riemannian tensor of the manifold M^k by $R_{\alpha_1, \alpha_2, i_1, i_2}$. If k is even, form the tensor*

$$R^{\alpha_1, \alpha_2}_{\ldots, i_1, i_2} R^{\alpha_3, \alpha_4}_{\ldots, i_3, i_4} \ldots R^{\alpha_{k-1}, \alpha_k}_{\ldots, i_{k-1}, i_k}$$

Alternating it over all indices $i_1, i_2, \ldots, i_k$, we obtain a new tensor determined by one scalar. Denote by G some component of this tensor different

from zero. G is a number function of the point of the manifold M^k. It turns out that integrating the number G over the manifold M^k we obtain its Eulerian characteristic multiplied by a certain constant $c(k)$ depending only on the dimensionality k. Let now $r=2s\leqslant k$ be a number divisible by four. Form the tensor

$$R^{a_1}_{\cdot\cdot, a_2, i_1, i_2} R^{a_2}_{\cdot\cdot, a_3, i_3, i_4} R^{a_3}_{\cdot\cdot, a_4, i_5, i_6} \cdots R^{a_s}_{\cdot\cdot, a_1, i_{r-1}, i_r}$$

Alternating this tensor over all indiees $i_1, i_2, \ldots, i_r$, we obtain a new tensor which we denote by P^r. If $r_1+r_2+\ldots+r_t\leqslant k$, then we denote the alternated or outer product of the tensors $P^{r_1}, P^{r_2}, \ldots, P^{r_t}$ by $P^{r_1, r_2, \ldots, r_t}$. It turns out that all skew-symmetric fields thus obtained on M^k are closed and up to homologies are topologic invariants of the manifold M^k. In other words, the integral of the field $P^{r_1, r_2, \ldots, r_t}$ over an arbitrary cycle Z is independent of the manner in which M^k is included into R^{k+l}, but depends only on the numbers $r_1, r_2, \ldots, r_t$ and the class of homologies of the cycle Z.

Proof. To every point $u\in M^k$ we correlate the plane $T(u)\in H(k, l)$ parallel to the plane tangent to M^k at the point u. Let q be an arbitrary point from M^k, F a certain skew-symmetric multilinear form of degree $r\leqslant k$ defined at the point $T(q)$ of the manifold $H(k, l)$, and d^pu, $(p=1, 2, \ldots, r)$, a system of r variable vectors defined at the point q of the manifold M^k. Correlate now to the form F a form $T^{-1}F$ defined at the point $q\in M^k$ by setting

$$T^{-1}F=F(T(d^1u),\ T(d^2u),\ \ldots,\ T(d^ru))\text{-}$$

Thus, to every field V defined on $H(k, l)$ there corresponds, in virtue of the mapping T, the field $T^{-1}V$ on M^k. Let us show that to invariant fields constructed on $H(k, l)$ in Theorem 1 correspond fields indicated in Theorem 2. Thereby Theorem 2 will be in essence proved.

To this end we place the point o determining the manifold $H(k, l)$ in the point q, and take R^k_0 to be tangent to M^k at the point o. Thus, in R^{k+l} is chosen a certain Cartesian system of coordinates such that the first k-dimensional coordinate plane is tangent to the manifold M^k at the origin. If u is a certain point of M^k in the neighbourhood of q, and $u^1, u^2, \ldots, u^k$, $y_1, y_2, \ldots, y_l$ are its Cartesian coordinates in the chosen system, then $u^1, u^2, \ldots, u^k$ may be considered to be their functions, whereby is satisfied the condition:

$$\text{for } u^1=u^2=\ldots=u^k=0,\ \frac{dy_j}{du^i}=0\ (i=1, 2, \ldots, k;\ j=1, 2, \ldots, l) \qquad (2)$$

It is easily seen that the plane $T(u)$ has for its coordinates the elements of the matrix $\left\|\frac{dy_j}{du^i}\right\|$, and hence it follows that to the vector $du=(du^1, du^2, \ldots du^k)$ defined at the point $o\in M^k$ corresponds under the mapping T the vector $\left\|\frac{\partial^2 y_j(0)}{\partial u^i\,\partial u^\alpha}du^\alpha\right\|$ at the point R^k_0. Herein the derivatives are taken at the values $u^1=u^2=\ldots=u^k=0$. If now $d^1u, d^2u, \ldots, d^ku$ is a system of k variable vectors defined at the point $o\in M^k$, and $x^p=T(d^pu)$, then the expressions $x^{p,q}_{\alpha\beta}$ constructed in Theorem 1 obtain an easily computable form, which after alternating over the vectors x^p and x^q leads to the formula

$$x^{p,q}_{\alpha,\beta}-x^{q,p}_{\alpha,\beta}=\sum_{j=0}^{l}\left(\frac{\partial^2 y_j(0)}{\partial u^\alpha\,\partial u^\gamma}\,\frac{\partial^2 y_j(0)}{\partial u^\beta\,\partial u^\delta}-\frac{\partial^2 y_j(0)}{\partial u^\alpha\,\partial u^\delta}\,\frac{\partial^2 y_j(0)}{\partial u^\beta\,\partial u^\gamma}\right)d^pu^\gamma d^qu^\delta$$

Let us now compute the right-hand side of the obtained equality through the Riemannian tensor of the manifold M^k at the point o. It is easily seen

that the metric tensor of the manifold M^k is given in the neighbourhood of the point o by the formula

$$g_{\alpha,\beta} = \delta_{\alpha,\beta} + \sum_{j=1}^{l} \frac{\partial y_j}{\partial u^\alpha} \frac{\partial y_j}{\partial u^\beta} \tag{3}$$

$u^1, u^2, \ldots, u^k$ being independent variables. Hence we see that at the point o all derivatives of the metric tensor are equal to zero [cf. (2)], and consequently the Riemannian tensor at the point o is given by the formula

$$R_{\alpha\beta\gamma\delta} = \frac{\partial^2 g_{\alpha\gamma}}{\partial u^\beta \partial u^\delta} + \frac{\partial^2 g_{\beta\delta}}{\partial u^\alpha \partial u^\gamma} - \frac{\partial^2 g_{\beta\gamma}}{\partial u^\alpha \partial u^\delta} - \frac{\partial^2 g_{\alpha\delta}}{\partial u^\beta \partial u^\gamma} =$$

$$= -2 \sum_{j=1}^{l} \left(\frac{\partial^2 y_j(0)}{\partial u^\alpha \partial u^\gamma} \frac{\partial^2 y_j(0)}{\partial u^\beta \partial u^\delta} - \frac{\partial^2 y_j(0)}{\partial u^\alpha \partial u^\delta} \frac{\partial^2 y_j(0)}{\partial u^\beta \partial u^\gamma} \right)$$

[cf. (2), (3)]. From this formula the asserted proposition follows immediately.

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MECHANICS

ON THE INTEGRABILITY OF HAMILTON-JACOBI EQUATION IN QUASI-COORDINATES

By V. V. DOBRONRAVOV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9, X. 1943)

In one of our papers (1) we deduced the partial differential equation of Hamilton for mechanical systems in quasi-coordinates, and stated at the same time the corresponding generalization of Jacobi's theorem.

In another paper (2) we establish the important invariance property of the Hamilton-Jacobi equation in quasi-coordinates with respect to different systems of quasi-coordinates generated by one and the same basis.

From this invariance property follows that all criterions of integrability in quadratures of the usual Hamilton-Jacobi equation may be applied also to the Hamilton-Jacobi equation in quasi-coordinates, inasmuch as the method of quasi-coordinates leads in the end to certain equations in true coordinates.

The Hamilton-Jacobi equation in quasi-coordinates has the form

$$\frac{\partial V}{\partial t}+H\left[\left(\frac{\partial V}{\partial \pi_i}\right), q_i, t\right]=0 \tag{1}$$

where the quasi-coordinates π_i and the true coordinates q_i are connected by the relations

$$\frac{d\pi_\rho}{dt}\equiv \omega_\rho=\sum_{\mu=1}^{n}\alpha_{\rho\mu}q'_\mu \tag{2}$$

$$q'_\mu=\sum_{\lambda=1}^{n}\beta_{\lambda\mu}\omega_\lambda \tag{3}$$

The coefficients of the matrices of the transformations (2) and (3) satisfy the conditions

$$\sum_{\mu=1}^{n}\alpha_{\rho\mu}\beta_{\lambda\mu}=\delta_\rho^\lambda\begin{cases}=1 & \text{if } \lambda=\rho\\ =0 & \text{if } \lambda\neq\rho\end{cases} \tag{4}$$

Suppose now that the kinetic energy and the force function have the form (Stäckel's criterion)

$$T=\frac{D}{2}\sum_{\nu=1}^{n}\frac{q'^2_\nu}{\Phi_{1\nu}};\qquad U=\frac{\sum_{\nu=1}^{n}f_\nu(q_\nu)\Phi_{1\nu}}{D} \tag{5}$$

where

$$D = |\varphi_{ij}(q_j)| \tag{6}$$

is a determinant of the n-th order, each element of which depends respectively only on one variable q_j; the functions $\Phi_{1\nu}$ are the adjuncts of the elements of the first line of this determinant; $f_j(q_j)$ are arbitrary functions, each of one argument q_j.

Form now under these conditions the Hamiltonian function H and the Hamilton-Jacobi equation in quasi-coordinates, *i. e.* the equation (1).

We have

$$p_\nu = \frac{\partial T}{\partial \omega_\nu} = \sum_{i=1}^{n} \frac{\partial T}{\partial q_i'} \frac{\partial q_i'}{\partial \omega_\nu}$$

or, in virtue of (5) and (3)

$$p_\nu = D \sum_{i=1}^{n} \frac{q_i'}{\Phi_{1i}} \beta_{\nu i} \qquad (\nu = 1, 2, \ldots, n). \tag{7}$$

Hence, solving the equation (7) with respect to q_i', we have

$$q_i' = \frac{\Phi_{1i}}{D} \frac{\sum_{\nu=1}^{n} p_\nu B_{\nu i}}{B} \tag{8}$$

where by B is denoted the determinant of the matrix of the transformation (3), and by $B_{\nu i}$ the adjuncts of the elements of this determinant.

Substituting the expression (8) into the expression for the kinetic energy, we find

$$T = \frac{1}{2D} \sum_{\nu=1}^{n} \Phi_{1\nu} \left(\sum_{i=1}^{n} \frac{p_i B_{i\nu}}{B} \right)^2 \tag{9}$$

But in virtue of the fact that the matrices (2) and (3) are inverse to each other, we have

$$\frac{B_{\nu i}}{B} = \alpha_{\nu i}$$

and, consequently, the Hamiltonian function assumes the form

$$H = \frac{1}{2D} \sum_{\nu=1}^{n} \Phi_{1\nu} \left(\sum_{i=1}^{n} \alpha_{i\nu} p_i \right)^2 - \frac{\sum_{\nu=1}^{n} f_\nu(q_\nu) \Phi_{1\nu}}{2D} \tag{10}$$

In order to form the equation (1) we have to replace in the expression (10) all p_i by $\left(\frac{\partial V}{\partial \pi_i}\right)$, *i. e.* by the operators $\sum_{\mu=1}^{n} \frac{\partial V}{\partial q_\mu} \beta_{i\mu}$.

Effecting this substitution and taking into account the relation (4), we obtain the partial differential equation

$$\frac{\partial V}{\partial t} + \frac{1}{2D} \sum_{\nu=1}^{n} \left[\Phi_{1\nu} \left(\frac{\partial V}{\partial q_\nu} \right)^2 - 2\Phi_{1\nu} f_\nu(q_\nu) \right] = 0 \tag{11}$$

In virtue of scleronomeness of the system we may put $V = -ht + W$, and the equation (11) takes the form

$$\frac{1}{2D}\sum_{\nu=1}^{n}\left[\Phi_{1\nu}\left(\frac{\partial W}{\partial q_\nu}\right)^2 - 2\Phi_{1\nu}f_\nu(q_\nu)\right] = h \tag{12}$$

which may be integrated by separating the variables.

In fact, let us look for a complete integral of the equation (12) in the form

$$W = \sum_{j=1}^{n} W_j(q_j) \tag{13}$$

where each $W_j(q_j)$ is a function of one q_j only.

To obtain the functions $W_j(q_j)$ we observe, in the first place, that in virtue of (12) they must satisfy the equation

$$\sum_{j=1}^{n}\Phi_{1j}\left(\frac{dW_j}{dq_j}\right)^2 = 2\left[\sum_{j=1}^{n} f_j\Phi_{1j} + hD\right] = 0 \tag{14}$$

Besides, we have the following identities

$$\sum_{\nu=1}^{n}\varphi_{j\nu}\Phi_{1\nu} = D\delta_j^1 \qquad (j = 1, 2, \ldots, n) \tag{15}$$

Substituting into (14) instead of D its expression from (15) and adding to it all remaining $(n-1)$ identities multiplied by arbitrary constants $2a_1, 2a_2, \ldots, 2a_{n-1}$, we obtain, after collecting terms with equal Φ_{1j}, the equation

$$\sum_{j=1}^{n}\Phi_{1j}\left\{\left(\frac{dW_j}{dq_j}\right)^2 - 2\left[f_j + h\varphi_{1j}(q_j) + \sum_{i=2}^{n} a_{i-1}\varphi_{ij}(q_j)\right]\right\} = 0 \tag{16}$$

Since each curved bracket occurring as a factor of Φ_{1j} depends only on one of the variables q_j, which are independent, we may write down the n equalities

$$\left(\frac{dW_j}{dq_j}\right)^2 - 2\left[f_j + h\varphi_{1j}(q_j) + \sum_{i=2}^{n} a_{i-1}\varphi_{ij}(q_j)\right] = 0 \qquad (j = 1, 2, \ldots, n) \tag{17}$$

from which we find the W_j by quadratures, and so obtain the complete integral of the equation (1) with the necessary number of arbitrary constants, one of which will occur additively.

Thus, the integrability in quadratures of the Hamilton-Jacobi equation, composed by the method of quasi-coordinates, is proved in the case under consideration.

As to the Liouville's criterion [4], it is, as is known, a particular case of Stäckel's criterion.

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PHYSIQUE MATHÉMATIQUE

SUR LE NOMBRE DES CYCLES LIMITES AU VOISINAGE D'UN FOYER

Par N. OTROKOV

(Présenté par N. D. Papalexi, de l'Académie, le 30. IX. 1943)

J'étudie dans cette Note l'équation différentielle

$$\frac{dy}{dx}=\frac{x+ay+\sum_{i+k=2}^{N} b_{ik}x^iy^k}{ax-y+\sum_{i+k=2}^{N} a_{ik}x^iy^k} \qquad (1)$$

(N est un nombre positif fixe. α, a_{ik}, b_{ik} sont des nombres réels quelconques), et je démontre que pour toutes les valeurs positives de δ on peut toujours indiquer des valeurs $\alpha=\alpha^*$, $a_{ik}=a_{ik}^*$, $b_{ik}=b_{ik}^*$ telles que l'équation (1) pour ces valeurs des coefficients admet à l'origine des coordonnées un point singulier du type de foyer, dans le voisinage δ duquel se trouvent $\frac{N^2+2N-4}{2}$ cycles limites, si N est pair, et $\frac{N^2+2N-9}{2}$ cycles limites, si N est impair, et d'ailleurs ces cycles entourent le foyer*.

§ 1. En introduisant dans (1) les coordonnées r et θ, désignons par $r=r\ (c, \theta, \alpha, a_{ik}, b_{ik})$ la solution de (1) qui prend pour $\theta=0$ la valeur $r=c$.

La détermination du nombre des cycles limites d'une équation du type (1) se trouvant dans un voisinage suffisamment petit de l'origine, se réduit à celle du nombre des zéros positifs suffisamment petits de la fonction holomorphe

$$\psi(c, \alpha, a_{ik}, b_{ik})=r(c, 2\pi, \alpha, a_{ik}, b_{ik})-c=\sum_{j=1}^{\infty} g_jc^j.$$

Lemme 1. Supposons que pour certaines valeurs déterminées $a_{ik}=a_{ik}^*$, $b_{ik}=b_{ik}^*$ la fonction $\psi(c, 0, a_{ik}^*, b_{ik}^*)$ n'est pas identiquement nulle, et l'équation $\psi(c, 0, a_{ik}^*, b_{ik}^*)=0$ possède k racines positives simples inférieures à un certain $\delta>0$.

* A ce que je sache, le nombre des cycles limites dans le voisinage d'un foyer n'a été étudié jusqu'à présent que pour la valeur particulière $N=2$. Grâce aux travaux de Kapteyn et Dulac, qui ont établi pour $N=2$ tous les cas de centres, la question a pu etre entièrement résolue pour $N=2$[1].

On peut alors indiquer pour tout $\varepsilon > 0$ une valeur $|\alpha^*| < \varepsilon$ telle que l'équation $\psi(c, \alpha^*, a^*_{ik}, b^*_{ik}) = 0$ possède au moins $k+1$ racines positives simples inférieures à δ.

Lemme 2. Supposons que les coefficients $g_j = g_j(\alpha, a_{ik}, b_{ik})$ satisfont aux trois conditions suivantes:

1) Il existe un système de valeurs

$$\alpha = 0, \quad a_{ik} = a^0_{ik}, \quad b_{ik} = b^0_{ik} \tag{2}$$

qui annule tous les g_j.

2) On peut indiquer m fonctions parmi les g_j, désignons les $g_{k_1}, g_{k_2}, \ldots, g_{k_m}$, et m coefficients $a_{i_p k_q}, b_{i_p k_q}$ tels que, les valeurs des autres coefficients tirées de (2) étant fixées: $a_{ik} = a^0_{ik}$, $b_{ik} = b^0_{ik}$ $(i \neq i_p,\ k \neq k_q)$, le système d'équations

$$x_1 = g_{k_1}, \quad x_2 = g_{k_2}, \ldots, x_m = g_{k_m}$$

soit résoluble par rapport aux $a_{i_p k_q}$, $b_{i_p k_q}$ au voisinage de leurs valeurs tirées de (2), et que ces solutions soient développables en séries convergentes ordonnées suivant les puissances positives entières des variables auxiliaires x_l.

3) Si l'on fait dans g_j la substitution des développements des $a_{i_p k_q}$, $b_{i_p k_q}$ en séries de puissance des x_l, et des valeurs tirées de (2) des autres a_{ik}, b_{ik}, les termes linéaires par rapport aux x_l de ceux des g_j pour lesquels $k_s < j < k_{s+1}$ contiennent seulement $x_1, x_2, \ldots, x_s$.

Dans ce cas, si petits que soient $\varepsilon > 0$ et $\delta > 0$, on peut toujours indiquer des valeurs $\alpha = 0$, $a_{i_p k_q} = a^*_{i_p k_q}$, $b_{i_p k_q} = b^*_{i_p k_q}$, vérifiant les inégalités

$$|a^*_{i_p k_q} - a^0_{i_p k_q}| < \varepsilon, \quad |b^*_{i_p k_q} - b^0_{i_p k_q}| < \varepsilon \tag{3}$$

telles que pour ces valeurs des $a_{i_p k_q}$, $b_{i_p k_q}$ et pour les valeurs des autres a_{ik}, b_{ik} tirées de (2), l'équation $\psi(c, 0, a_{ik}, b_{ik}) = 0$ ne se réduit pas à une identité, et admet au moins $m-1$ racines positives inférieures à δ.

§ 2. Théorème. *Pour chaque entier $N > 0$ on peut indiquer un système des nombres $0, a^0_{ik}, b^0_{ik} (i, k = 2, 3, \ldots, N)$ tel que:*

1) *pour $\alpha = 0$, $a_{ik} = a^0_{ik}$, $b_{ik} = b^0_{ik}$ l'équation (1) possède à l'origine de coordonnées un point singulier du type «centre»:*

2) *si petits que soient $\delta > 0$, $\varepsilon > 0$, on peut toujours indiquer des valeurs $\alpha = \alpha^*$, $a_{ik} = a^*_{ik}$, $b_{ik} = b^*_{ik}$ vérifiant les inégalités:*

$$|a^*_{ik} - a^0_{ik}| < \varepsilon, \quad |b^*_{ik} - b^0_{ik}| < \varepsilon, \quad |\alpha^*| < \delta$$

telles que l'équation différentielle (1) possède dans le voisinage δ de l'origine de coordonnées $\frac{N^2+2N-4}{2}$ cycles limites pour N pair, et $\frac{N^2+2N-9}{2}$ cycles limites pour N impair.

Démonstration. Posons $\alpha = 0$ et introduisons la notation

$$a_{n0} = \alpha_{n0}, \quad b_{n0} = \beta_{n0}, \quad b_{0n} = \alpha_{-1,n+1}$$
$$a_{n-k,k} + b_{n-k+1,k-1} = \alpha_{n-k,k}, \quad b_{n-k,k} - a_{n-k+1,k-1} = \beta_{n-k,k}$$
$$(n = 2, \ldots, N;\ k = 1, 2, \ldots, n) \tag{4}$$

Remarquons que pour les valeurs de $a_{ik}, b_{ik} (\alpha = 0)$ pour lesquelles tous les $\alpha_{ik} = 0$ (les β_{ik} peuvent être quelconques) l'équation (1) possède à l'origine de coordonnés un point du type «centre», donc pour ces valeurs

$$\psi(c, 0, a_{ik}, b_{ik}) \equiv 0.$$

1. N pair. En posant $\beta_{ik}=0$ pour $i+k<N$ et en laissant les β_{ik} encore arbitraires pour $i+k=N$, nous considérerons les α_{ik} comme variables, et en introduisant des variables auxiliaires x_{n_p} nous considérons le système d'équations

$$g_{n+(2p+1)(N-1)}=x_{n_p}\begin{pmatrix}n=2,4,\ldots,N\\ p=0,1,\ldots,n+1\end{pmatrix}$$

$$g_{n+2p(N-1)}=x_{n_p}\begin{pmatrix}n=3,5,\ldots,N-1\\ p=0,1,\ldots,n+1\end{pmatrix} \qquad (5)$$

Déterminons le rang de la matrice des termes linéaires en α_{ik} dans les premiers membres des équations (5). Ces termes linéaires sont de la forme

$$\sum_{k=0}^{n+1}\alpha_{n-k,k}A^{(p)}_{n-k,k} \quad (n=2,3,\ldots,N)$$

où

$$A^{(p)}_{n-k,k}=\frac{(-1)^p}{2\pi}\int_0^{2\pi}\cos^{n-k+1}\theta\sin^k\theta\,(A_N)^p\,d\theta$$

et

$$A_N=\sum_{k=0}^{\frac{N-2}{2}}\{\beta_{N-2k,2k}[\cos^{N-2k+1}\theta\sin^{2k}\theta+(-1)^{N/2+k+1}\cos\theta\sin^N\theta]+$$
$$+\beta_{N-2k+1,2k+1}[\cos^{N-2k}\theta\sin^{2k+1}\theta+(-1)^{N/2+k+1}\sin^{N+1}\theta]\}$$

et peuvent être divisés en $N-1$ groupes, dans chacun desquels n a la même valeur et p prend les valeurs $0, 1, 2, \ldots, n+1$.

Le rang de la matrice des termes linéaires en α_{ik} des équations (5) est égal à la somme des rangs des déterminants $D_{n+2}\,(n=2,3,\ldots,N)$, où

$$D_{n+2}=|A^{(p)}_{n-k,k}|$$

et $p=2i+1$, si n est pair, $p=2i$, si n est impair, et $i, k=0, 1, 2, \ldots, n+1$.

Dans le cas de n pair, l'étude dans les éléments de D_{n+2} des termes seulement qui contiennent $\beta_{1,N-1}$ et $\beta_{n,N-n}$ démontre que D_{n+2} possède un mineur d'ordre n, non identiquement nul par rapport aux $\beta_{ik}\,(i+k=N)$. Dans le cas de n impair, l'étude des termes contenant $\beta_{1,N-1}$ et $\beta_{n-1,N-n+1}$ des éléments de D_{n+2}, montre que D_{n+2} possède un mineur d'ordre $n+1$, non identiquement nul par rapport aux $\beta_{ik}\,(i+k=N)$.

De là résulte qu'on peut toujours choisir des valeurs β_{ik}, $i+k=N$, telles que le rang de la matrice des termes linéaires par rapport aux α_{lk} dans les premiers membres des équations (5) soit égal à $\frac{N^2+2N-6}{2}$.

2. N impair. Posons $\beta_{ik}=0$ pour $i+k\neq N-1$, les β_{ik} pour $i+k=N-1$ resteront encore indéterminés. Posons d'ailleurs $\alpha_{1,1}=\alpha_{1,2}=\alpha_{-1,3}=0$ et considérons les autres α_{ik} comme des variables; étudions le système d'équations

$$g_{n+(2p+1)(N-2)}=x_{n_p}\begin{pmatrix}n=2,4,\ldots,N-1\\ p=0,1,\ldots,n+1\end{pmatrix}$$

$$g_{n+2p(N-2)}=x_{n_p}\begin{pmatrix}n=3,5,\ldots,N\\ p=0,1,\ldots,n+1\end{pmatrix} \qquad (5')$$

Des raisonnements analogues à ceux qui ont été faits pour le cas de N pair démonterent qu'on peut toujours choisir des valeurs de $\beta_{ik}\,(i+k=N-1)$ tel-

les que le rang de la matrice des termes linéaires par rapport aux α_{ik} dans les premiers membres des équations (5′) soit égal à $\frac{N^2+2N-11}{2}$.

Il résulte de ce qui précède qu'en posant $m=\frac{N^2+2N-6}{2}$ pour N pair, et $m=\frac{N^2+2N-11}{2}$ pour N impair, on peut toujours choisir m quantités $\alpha_{i_s k_s}$, et supposer les autres α_{ik} nuls, et m équations parmi (5) ou bien (5′) de telle façon que pour certaines valeurs déterminées $\beta_{ik}=\beta_{ik}^0$ le jacobien du système de m équations choisies par rapport aux $\alpha_{i_s k_s}$ soit non nul. En tenant compte de la relation entre les α_{ik}, β_{ik} et les a_{ik}, b_{ik} exprimée par les équations (4) et des propriétés connues des fonctions g_j, on voit facilement que les fonctions g_j satisfont aux conditions du lemme 2.

D'ailleurs, dans notre cas les valeurs de a_{ik}^0, b_{ik}^0 dont il est question dans la condition 1) de ce lemme, sont telles que $\alpha_{ik}=0$, $\beta_{ik}=\beta_{ik}^0$ $(\alpha=0)$, et le nombre m a la valeur précédemment indiquée.

Pour démontrer le théorème, il suffit maintenant d'appliquer le lemme 1.

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LITTÉRATURE CITÉE

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PHYSICS

DIFFUSION OF LIGHT THROUGH SCATTERING MEDIUM OF LARGE OPTICAL THICKNESS

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The subject of the present paper is the diffusion of light through a turbid medium of large optical thickness τ_0, composed of plane-parallel layers. Let the outward radiation enter the medium through one of its plane boundaries. In this case part of the radiation will be sent back as a diffuse reflected light, and the rest, upon diffusion through the medium, will get out across the other boundary. Take the latter for the starting point in measuring the optical thicknesses. Then the optical thickness at that boundary is $\tau=0$, and at the boundary through which light enters the medium, $\tau=\tau_0$. With τ_0 increasing to infinity the flow that has traversed the medium will diminish gradually. But the relative distribution of intensities of the radiation crossing the boundary $\tau=0$ in the different directions will tend to a certain limit which does not depend upon the direction of the original incident rays. By increasing in a definite manner the intensity of the rays falling from outside upon the boundary $\tau=\tau_0$, we can reach such a state of affairs that the total flow of the radiation that has traversed the medium will remain constant as τ_0 grows. Under these conditions the intensity of the emerging radiation in every direction will tend to a certain limit for $\tau_0\to\infty$. This intensity will be a function of the angle between the emerging radiation and the outward normal. The cosine of this angle we denote by η, and the limiting intensity sought-for, by $u(\eta)$. Our problem is to find the function $u(\eta)$. This problem is equivalent to that where a turbid medium fills up a half-space, the source of radiation is at infinity, and we have to find the distribution of the rays emerging from the boundary according to direction.

Let us note, moreover, that $u(\eta)$ is determined only up to a constant factor.

In solving this problem we will apply the method used in our earlier works (1). It consists in adding a layer of small thickness $\Delta\tau$.

The value of τ_0 being very large, the addition of $\Delta\tau$ will not affect the relative distribution of intensities over the directions of the emerging rays, though it will diminish (if there is absorption in the medium as well as cattering) the total flux of emerging light. Therefore, upon the addition of the layer $\Delta\tau$, the distribution will be determined by the same function $u(\eta)$ multiplied by the factor $1-k\Delta\tau$, where the constant k is to be determined. On the other hand, we can determine the emerging intensity by calculating all changes consequent upon the addition of the layer $\Delta\tau$.

In the first place, the layer $\Delta\tau$ will weaken the rays travelling in the direction η by as much as $1-\frac{\Delta\tau}{\eta}$ times. On the other hand, the radiation coming from the former boundary $\tau=0$ is scattered by this layer in the direction η.

This scattered radiation is defined by the integr

$$\frac{\lambda\Delta\tau}{4\pi\eta}\int\int x(\eta,\varphi;\ \eta',\varphi')\ u(\eta')d\eta'\,d\varphi'$$

where $x(\eta, \varphi; \eta', \varphi')$ is the indicatrix of scattering. It is a function of the angle between the directions η, φ and η', φ'. The parameter λ is the ratio of the scattering coefficient to the total coefficient of extinction.

Moreover, part of the radiation scattered by the layer $\Delta\tau$ falls back on the former boundary $\tau=0$ and undergoes a diffuse reflection from it. The intensity of this reflected radiation is easily seen to be determined by

$$\frac{\lambda\Delta\tau}{4\pi^2}\int\int r(\eta,\varphi;\ \eta';\ \varphi')\frac{d\eta'}{\eta'}\,d\varphi'\int\int x(\eta',\varphi';-\eta'',\varphi'')\,u(\eta'')\,d\eta''d\varphi''$$

where $r(\eta, \varphi; \eta', \varphi')$ is a function characterizing the diffuse reflection by a medium of infinite optical thickness. Its exact definition is as follows: if $I(\eta, \varphi)$ is the intensity of the radiation incident upon the boundary in the different directions η, φ, then the intensity $I_1(\eta, \varphi)$ of the diffuse reflected light is

$$I_1(\eta,\varphi)=\frac{1}{\pi}\int\int r(\eta,\varphi;\ \eta',\varphi')\,I(\eta',\varphi')\,d\eta'd\varphi'$$

Now we can equate the two expressions for the intensity obtained after the addition of the layer $\Delta\tau$

$$u(\eta)(1-k\Delta\tau)=u(\eta)\left(1-\frac{\Delta\tau}{\eta}\right)+\frac{\lambda\Delta\tau}{4\pi\eta}\int\int x(\eta,\varphi;\ \eta',\varphi')\,u(\eta')\,d\eta'd\varphi'+$$
$$+\frac{\lambda\Delta\tau}{4\pi^2}\int\int r(\eta,\varphi;\ \eta',\varphi')\frac{d\eta'}{\eta'}d\varphi'\int\int x(\eta',\varphi';-\eta'',\varphi'')\,u(\eta'')\,d\eta''\,d\varphi''$$

or

$$u(\eta)\left(\frac{1}{\eta}-k\right)=\frac{\lambda}{4\pi\eta}\int\int x(\eta,\varphi;\ \eta',\varphi')\,u(\eta')\,d\eta'd\varphi'+$$
$$+\frac{\lambda}{4\pi^2}\int\int r(\eta,\varphi;\ \eta',\varphi')\frac{d\eta'}{\eta'}d\varphi'\int\int x(\eta',\varphi';-\eta'',\varphi'')\,u(\eta'')\,d\eta''\,d\varphi''\qquad(1)$$

As u is independent of the azimuth, and the functions x and r are related to the arguments φ and φ' only through the difference $\varphi-\varphi'$, the integrals

$$\frac{1}{2\pi}\int_0^{2\pi}x(\eta,\varphi;\ \eta',\varphi')\,d\varphi'=p(\eta,\eta')\qquad(2)$$

$$\frac{1}{2\pi}\int_0^{2\pi}r(\eta,\varphi;\ \eta',\varphi')\,d\varphi'=\frac{\lambda}{4}\Phi(\eta,\eta')\qquad(3)$$

do not depend upon the azimuth φ(2).

We may now write (1) in the form

$$u(\eta)(1-k\eta)=\frac{\lambda}{2}\int_0^1 p(\eta,\eta')\,u(\eta')\,d\eta'+\frac{\lambda^2}{4}\int_0^1\Phi(\eta,\eta')\frac{d\eta'}{\eta'}\int_0^1 p(\eta',-\eta'')\,u(\eta'')\,d\eta''\qquad(4)$$

Introducing the kernel

$$K(\eta,\eta')=p(\eta,\eta')+\frac{\lambda}{2}\int_0^1\Phi(\eta,\eta'')\,p(\eta'',-\eta')\frac{d\eta''}{\eta''}\qquad(5)$$

we, instead of (4), obtain for $u(\eta)$ the integral equation

$$u(\eta)(1-k\eta)=\frac{\lambda}{2}\int_0^1 K(\eta,\eta')\,u(\eta')\,d\eta'.\qquad(6)$$

The constant k should be so chosen as to secure a solution different from zero for a given λ.

In the previous work ([3]) we have established that the function $\Phi(\eta, \eta')$ is expressed in the form

$$\Phi(\eta, \eta') = \sum (-1)^i x_i \frac{\psi_i(\eta)\,\psi_i(\eta')}{\frac{1}{\eta}+\frac{1}{\eta'}} \tag{7}$$

where $\psi_i(\eta)$ are auxiliary functions determined by the system of equations

$$\psi_i(\eta) = P_i(\eta) + \frac{\lambda}{2}\,\eta \sum_j (-1)^{i+j} x_j \psi_j(\eta) \int_0^1 \frac{\psi_j(\eta')\,P_i(\eta')\,d\eta'}{\eta+\eta'} \tag{8}$$

where x_j are coefficients of the expansion of the indicatrix in Legendre's polynomials. On the other hand, $p(\eta, \eta')$ is representable by Legendre's polynomials in the form

$$p(\eta, \eta') = \sum_i x_i P_i(\eta) P_i(\eta') \tag{9}$$

Substituting (9) in (5) we find

$$K(\eta, \eta') = \sum x_i P_i(\eta') \left\{ P_i(\eta) + \frac{\lambda}{2} (-1)^i \int_0^1 \Phi(\eta, \eta'') P_i(\eta'') \frac{d\eta''}{\eta''} \right\}$$

or, by virtue of (7) and (8),

$$K(\eta, \eta') = \sum x_i P_i(\eta') \psi_i(\eta) \tag{10}$$

Substituting (10) in (6), we obtain

$$u(\eta) = \frac{\lambda}{2} \sum \frac{x_i \psi_i(\eta)}{1-k\eta} \int_0^1 P_i(\eta')\,u(\eta')\,d\eta'$$

Let us denote

$$\int_0^1 u(\eta)\,P_i(\eta)\,d\eta = c_i$$

Then

$$u(\eta) = \frac{\lambda}{2} \sum \frac{x_i c_i \psi_i(\eta)}{1-k\eta} \tag{11}$$

Multiplying by $P_j(\eta)$ and integrating, we get

$$c_j = \frac{\lambda}{2} \sum_i a_{ij} x_i c_i \tag{12}$$

where

$$a_{ij} = \int_0^1 \frac{\psi_i(\eta)\,P_j(\eta)\,d\eta}{1-k\eta} \tag{13}$$

Thus, the distribution of radiation according to directions, which it is our aim to find, is determined by means of (11) through the auxiliary functions $\psi_i(\eta)$, and the coefficients c_i are determined from the homogeneous system (12).

By way of example let us consider two simple cases.

1. Spherical Indicatrix of Scattering. All x_i are zeros except $x_0 = 1$. Formula (11) reduces to

$$u(\eta) = \frac{c\psi_0(\eta)}{1-k\eta}. \tag{14}$$

$\psi_0(\eta)$ is tabulated by the author for different λ. Instead of (12) we have in this case

$$1 = \frac{\lambda}{2} a_{00}$$

According to (13) this means

$$1 = \frac{\lambda}{2} \int_0^1 \frac{\psi_0(\eta)\, d\eta}{1 - k\eta} \qquad (15)$$

It may be shown ([3]) that the equation (14) is satisfied if k is such that

$$\lambda = \frac{1}{2k} \ln \frac{1+k}{1-k} \qquad (16)$$

2. Indicatrix of Scattering of the Type $x(\eta, \varphi; \eta', \varphi') = 1 + x_1 \cos(\eta, \varphi; \eta', \varphi')$. Let us take the case $\lambda = 1$. With this type of indicatrix $\psi_0(\eta)$ and $\psi_1(\eta)$ are, in general, distinct from zero. But with $\lambda = 1$ we have $\psi_1(\eta) = 0$, as has been shown by the author earlier ([3]). Therefore (11) is again reduced to (14), and the relation between λ and k remains in the form (16). But then $\lambda = 1$ implies $k = 0$. Therefore

$$u(\eta) = c\psi_0(\eta) \qquad (17)$$

The remarkable conclusion to be drawn from this is that in the case of pure scattering the relative distribution of intensities over the directions of emerging radiation for every indicatrix of the type $1 + x_1 \cos(\eta, \varphi; \eta', \varphi')$ is the same as for the spherical indicatrix.

This conclusion bears directly upon the problem of the distribution of light over a sky of uniform cloudiness. If the albedo of the Earth surface is small enough, the uniform layer of clouds may be looked upon as a pure scattering layer of large optical thickness, illuminated by direct sun rays from only one side. The radiation falling on the lower boundary of the layer of clouds may be neglected. Therefore the distribution of intensities of the diffusely transmitted light of the Sun with regard to direction (*i. e.* according to the zenith distance) should be determined by formula (11).

For clouds the form of the scattering indicatrix is unknown to us. But it is the extent to which it is drawn out that is of basic importance for the problem of brightness distribution over the directions of transmitted radiation. And the elongation of the indicatrix of scattering is characterized by the very coefficient x_1 of its development in Legendre's polynomials. If therefore the indicatrix be represented approximately in the form

$$1 + x_1 \cos(\eta, \varphi; \eta', \varphi')$$

the inaccuracy of the obtained value for brightness distribution over a cloudy sky (caused by omitting the higher terms of the expansion) is not likely to be very great.

Taking this and the natural fluctuations due to the unhomogeneity of the cloud layer into consideration, we must expect the brightness of a uniformly overcast sky in the absence of snow-cover and for any elongation of the scattering indicatrix to be determined by the zenith distance according to formula (17).

This formula gives

$$\frac{\psi_0(1)}{\psi_0(0)} = 2.9$$

as the brightness ratio of zenith to horizon.

Numerous observations of the distribution of brightness over a cloudy sky have been made at Elabuga by E. N. Yustova, of the local branch of the Leningrad State University. An average with respect to azimuth, derived from 14 days of observation, gives a value of 3.0 for the brightness ratio between the zenith and horizon. This perfectly agrees with the theoretical forecast.

It has been shown in my work that formula (17) also determines the distribution of brightness over the solar disc in the total light. Thus, if the terms of the second and higher power in the expansion of the indicatrix in Legendre's polynomials are neglected, the distribution of brightness over a uniformly overcast sky in the absence of snow-cover should coincide with that over the solar disc.

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PHYSICS

BIREFRINGENCE IN THE FLOW OF ISOTROPIC LIQUID *p*-AZOXYANISOL

By NIKITA TOLSTOY

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The isotropic liquid phase of substances giving an anisotropic liquid phase (nematic liquids) has been scantly investigated. However, at a temperature exceeding by several degrees the point of transition from the anisotropic

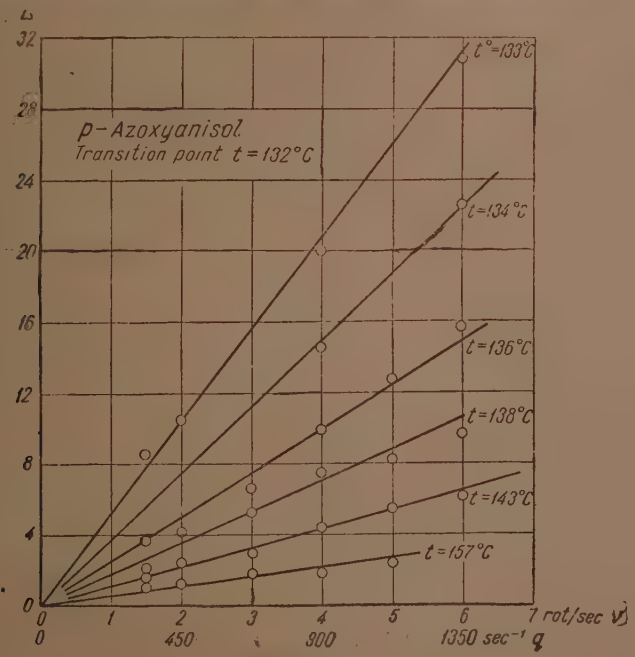

Fig. 1

liquid phase to an isotropic liquid, interesting phenomena are observed, which consist in a rapid growth of magnetic birefringence (1) with decreasing temperature and in the growth of birefringence in the flow. The latter phenomenon has been described by Vorländer and Walter (2) only qualitatively and without measuring the extinction angle, *i. e.* the angle formed by the main optical axis of the anisotropy with the direction of the flow.

The present author has carried out quantitative measurements of birefringence and of the extinction angle in a flow of *p*-azoxyanisol within a range of tem-

peratures from 132° (transition point) to 157°C. Fig. 1 shows the dependence of birefringence upon the velocity gradient in the flow. The phase difference (Δ) of the ordinary and extraordinary ray is plotted in degrees on the axis of ordinates; the abscissae are the rotation number (ν) of the apparatus' rotor in seconds, and the velocity gradient. The curves bring out: 1) the linear growth of birefringence with the gradient (*i. e.* absence of saturation and the possibility of determining Maxwell's constant); 2) the growth of that constant with temperature. This growth is expressed in arbitrary units in Fig. 2. For the sake of comparison the values of the Cotton-Mouton constant as measured by Zadoc-Kahn are plotted in the same figure on a suitably chosen scale for the same range of temperature. The likeness of the temperature dependence for the two effects makes one to think that the reason for their increase does not lie in the variation of some specific (magnetic or hydrodynamic) properties of the liquid, but rather in the change of the intermolecular structure.

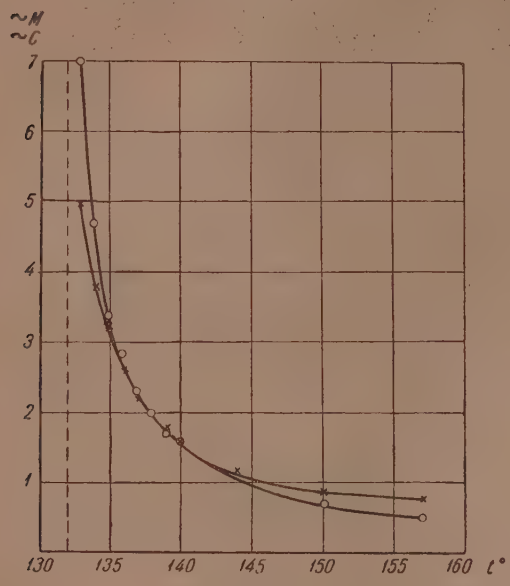

Fig. 2

Fig. 3 gives the dependence of the extinction angle χ upon the rotation number of the rotor in seconds, as established in a series of measurements carried out at different temperatures and under different conditions of observation (the extinction angle was measured in the laminar flow as well as in the turbulent; in the latter case the laminar layers adjoining the walls of the vessel were investigated; the gradient of velocity in these layers is rather high). Within the experimental error the angle with the direction of the flow is 45° under all conditions.

The sign of birefringence is positive in Maxwell's effect as well as in Cotton-Mouton's.

The measurements were conducted in the cylindrical apparatus of Couette with 0.63 and 1.3 mm slits; the path length was 17 mm. In measuring birefringence Senarmont's method was made use of (with a $\lambda/4$ plate). Both the liquid and the apparatus were carefully cleaned from dust (so that a half-shade plate could be used). An overglowed cinema lamp (50 W) served as a source of light. The substance was a dark-orange colour.

If one believes with Cotton [3], Kast [4] and others that above the transition temperature the molecules of the liquid continue to be united in groups, each as a whole orientated by the action of external fields but no longer interacting mutually, as they did in the anisotropic phase, then our results allow to determine the shape of these formations and estimate their maximum and minimum size.

Let us use for this purpose the theory developed by Stuart and Peterlin [5]. Suppose our groups to be as large as to permit one to speak of a refraction index along and across the group. The shape of the group we shall approximate to an ellipsoid of revolution. We assume that the ellipsoids are not interacting, nor subject to deformation, and are surrounded by an amorphous liquid whose birefringence would be small but for the presence of the groups (as is the case with all molecular inviscid liquids). We have then the following relations

$$M = \frac{\Delta n}{c \cdot q \cdot n \cdot \eta} = \frac{2\pi}{15 n^2} \frac{\varphi}{kT} (g_1 - g_3) \frac{b}{[p]} \qquad (1)$$

$$\frac{\pi/4-\chi}{q\eta}=\frac{1}{12}\frac{\varphi}{kT}\frac{1}{[p]} \qquad (2)$$

Here M is Maxwell's constant; c, concentration of groups in the liquid; q, gradient; n, mean index of refraction; φ, volume of a group; g_1-g_2, factor of anisotropy, varying with the shape of the group and the refraction indices along and across the group axis; k, Boltzman's constant; T, temperature; $[p]$, a function of the ratio between the lengths of the ellipsoid axes; b, another function of this ratio; η, viscosity of the medium surrounding the groups.

If the concentration of the groups in the medium were known, φ and p (the volume and shape of the ellipsoid representing the group) could be de-

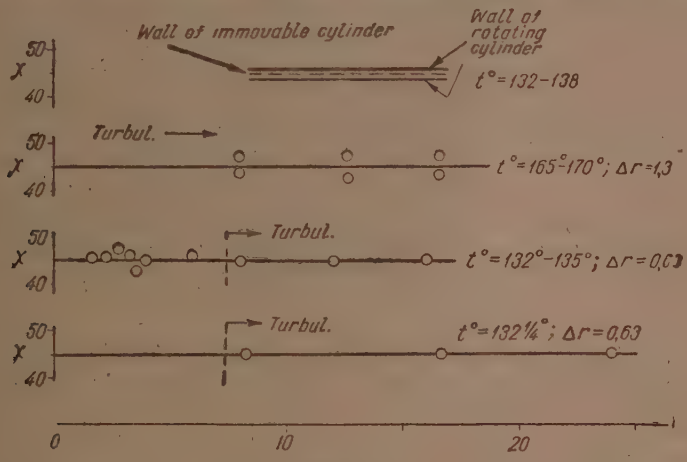

Fig. 3

termined from (1) and (2). Though we do not know the value of c, we can use the equation

$$C=\frac{\Delta n}{cnH^2}=\frac{2\pi}{15n^2}\frac{\varphi}{kT}(g_1-g_2)(\bar{g}_{m_1}-\bar{g}_{m_2}) \qquad (3)$$

where C is Cotton-Mouton's constant; H is the force of magnetic field; $\bar{g}_{m_1}-\bar{g}_{m_2}$, the factor of magnetic anisotropy which with sufficient accuracy (5) can be taken to be equal to

$$\bar{g}_{m_1}-\bar{g}_{m_2}=\frac{1}{4\pi}(\mu_1-\mu_2)=\varkappa_1-\varkappa_2 \qquad (4)$$

where $\mu_{1,2}$ is magnetic permeability; $\varkappa_{1,2}$, magnetic susceptibility along and across the axis of the group. From (1), (3) and (4) we get

$$\frac{b}{[p]}=\frac{(\varkappa_1-\varkappa_2)H^2}{q\eta}\frac{\Delta n_m}{\Delta n_c} \qquad (5)$$

where Δn_m and Δn_c is birefringence in the flow and in the magnetic field, respectively.

Let us substitute the value of Δn_m, Δn_c and η (6) for 134 and 154°C, taking 132°C for the point of transition to isotropic phase. The corresponding data are given in the annexed table.

Put, moreover, $\varkappa_1-\varkappa_2=2.4\times10^{-7}$ [see, e. g. (7)], $q=900$ sec.$^{-1}$, $H=33900$ Gauss. We obtain

$$\frac{b}{[p]}=\frac{p^2-1}{p^2+1}:\frac{p^2}{4(p^4-1)}\left[-1+\frac{2p^2-1}{2p\sqrt{p^2-1}}\log\frac{p+\sqrt{p^2-1}}{p-\sqrt{p^2-1}}\right]=$$
$$=\begin{cases}2.9 \text{ for } t=134°\text{C}\\ 4.8 \text{ for } t=154°\text{C}\end{cases}$$

Temperature (degrees above transition point)	Gradient 900 sec.$^{-1}$		Viscosity $\eta \times 10^3$	Maxwell constant $M \times 10^8$	Magnetic field $H = 33900$ Gauss		Cotton-Mouton's constant $C \times 10^{15}$
	path difference of rays, in degrees	difference of diffraction indices $\Delta n_m \times 10^6$			path difference of rays, in degrees	difference of diffraction indices $\Delta n_c \times 10^6$	
1°	20.0	1.92	1.74	7.5	>160	>13.1	>7
2°	15.2	1.42	1.72	5.8	110	9.0	4.7
3°	12.6	1.21	1.69	4.9	80	6,5	3.4
4°	10.0	0.96	1.66	3.9	64	5.2	2.8
5°	8.4	0.81	1.64	3.4	54	4.4	2.3
6°	7.4	0.71	1.62	3.0	48	3.8	2.0
7°	6.5	0.62	1.59	2.7	41	3.3	1.7
8°	5.8	0.56	1.56	2.4	37	3.0	1.6
12°	4.2	0.40	1.46	1.9	25	2.0	1.1
18°	2.8	0.27	1.30	1.4	17	1.4	0.7
25°	2.2	0.21	>1.20	<1.2	12	1.0	0.5

For the length ratio of the ellipsoid axes we respectively obtain

$$p \cong \begin{cases} 1.4 \text{ for } t = 134°\text{C} \\ 1.7 \text{ for } t = 154°\text{C} \end{cases}$$

It will be obvious that the group uniting the molecules of *p*-azoxyanisol is elongated approximately 1.5 times in the axial direction of the molecules packed parallel into the group.

A small increase of this elongation we believe to lie within the error of measurement. Should, however, this increase really take place, it might be explained by the fact that as the temperature rises, the group unites an ever decreasing number of molecules, and the rod-like nature of the *p*-azoxyanisol molecule gains more weight in determining the shape of the group by its own form.

It should be noted that the expression $g_1 - g_2$ does not enter in the formula (5). This will mean that our result is not too closely connected with the views on the structure of the particle (group) lying at the basis of Peterlin-Stuart's theory.

From equation (2) we may find the largest possible average volume of the groups

$$\bar{v}_{\max} = \frac{12kT\,[p]\,\Delta\chi_{\max}}{q\eta}$$

Here $\Delta\chi$ is the greatest average deviation of the angle χ from 45°. In our case $\Delta\chi$ is determined by the limits of the error and we put it at 0.02 radians. The highest gradient q at which χ was observed made > 5400 sec.$^{-1}$ (these estimates have a wide margin, firstly, because in some sets of measurements we managed to determine χ with an accuracy of 0.01 radians, and secondly, because the average gradient in the laminar layer of the turbulent flow greatly exceeds the value obtained by dividing the linear velocity of the rotor by the width of the slit).

Substituting $[p] = 1.2$, $T = 406$°C, $\eta = 0.017$, we get

$$\bar{v}_{\max} < 2 \times 10^{-17}$$

(we do not take here into account the variation $[p], \eta$ with temperature, since we are to obtain a rough estimate; if the influence of temperature were allowed for, the difference would be no less than 20 per cent). This value of the groups is in no conflict with X-ray data for the isotropic phase of *p*-azoxy-

anisol, even if one assumes, as we do, that the molecules in the group are ordered as in the crystal.

Compare v_{max} with the average volume v_0 of the *p*-azoxyanisol molecule. From the molecular weight $M_0 = 258$ and density $\rho = 1.14$ we get $v_0 = 3.7 \times 10^{-22}$. From this it is obvious that a group cannot possibly unite more than a few tens of thousands of molecules.

If all groups were equal and had the maximum established size, their minimum volume, concentration for a temperature as high as 1° above the transition point would be

$$c_{min} = \frac{\Delta n_m 15nkT}{2\pi v_{max}(g_1 - g_2)\, q\eta \dfrac{b}{[p]}}$$

The value $g_1 - g_2$ can be estimated from the graphs referred to in a paper by Stuart and Peterlin ([8]). We shall obtain

$$c_{min} > 0.4 \text{ per cent.}$$

The order of the minimum average magnitude of the groups for the same temperature can be computed on the assumption that the whole substance is broken into groups, or that c is equal to 1. We have

$$\bar{v}_{min} = v_{max}\, c_{min} = \frac{\Delta n_m 15nkT}{2\pi (g_1 - g_2)\, q\eta \dfrac{b}{[p]}} = 8 \times 10^{-20}$$

i. e. a group should include no less than several hundred molecules. At high temperatures it will unite only some tens of molecules as a minimum. Comparing the Maxwell effect in *p*-azoxyanisol at high temperature ($M \approx 10^{-8}$) with the same effect in benzene at room temperature [$M_{C_6H_6} = 6.4 \times 10^{-11}$ ([9])], we see that there is nothing improbable about the order of magnitude in our estimate, though the estimates of v_{min} imply a strong deviation from the physical limits of applicability of the Peterlin and Stuart theory.

To end with, compare the dimensions of our group with those of the groups existing in anisotropic phases [«swarms» ([4],[10])]. The latter are evaluated as $10^{-13} - 10^{-14}$ cm^3 and hence exceed our groups $10^4 - 10^7$ times.

I am glad to acknowledge my gratitude to L. N. Fedotov for assistance in carrying out the experiments.

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GEOLOGY

ON A COMPARATIVE EVALUATION OF THE WATER CONTENT OF AQUIFERS

By K. I. MAKOV

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At the present time abundant material has been collected on the underground waters of individual hydrogeological provinces of the USSR.

If subjected to an adequate treatment, these data permit of an objective evaluation of the water content both in an aquifer (as a whole and in its separate portions) and in the hydrogeological provinces taken as a whole.

In this paper this question is discussed as related to the fissure waters which are especially widespread over the area of the European part of the USSR. The method suggested is, however, fully applicable to aquifers confined to sands.

In making a comparative evaluation of the water content of some particular wells it is not enough to compare their specific discharges, since in this case the values compared would not be univocal. The discharge of wells fed with fissure waters is intimately related to the depth of the well, since the inflows of water in the general case proceed from the whole area of the aquiferous rocks exposed by the well.

An objective evaluation of the water content of fractured rocks should therefore be made according to the values of the specific water output, which represents the specific discharges referred to unit area of the exposed water-bearing rocks in the well. An ideal determination of the specific water output in this case would be according to strata (horizons), the diameter of the wells being taken into consideration; however, in the majority of cases no data are available for this purpose, and the total specific water output is to be determined by dividing the specific discharge by the thickness of the water-bearing rocks exposed; thus, the value of the specific water output, proposed for an objective evaluation of the water content of fractured rocks, will be equal to the discharge of the well Q divided by the lowering of the water level upon pumping S and by the thickness of the water-bearing rocks H.*

Such a computation does not comprise the influence upon the inflows of water of the value of the surface area of crystalline rocks exposed in the well.

To make an objective evaluation of the water content of individual aquifers and of hydrogeological provinces as a whole, it would be appropriate to use a

* A general formula for determining the specific water output may be suggested in the following form:

$$q_1 = \frac{Q_1}{S_1\left(\pi d_1 H_1 + \frac{\pi d_1^2}{4}\right)}; \quad q_2 = \frac{Q_2}{S_2\left(\pi d_2 H_2 + \frac{\pi d_2^2}{4}\right)}; \ldots$$

$$\ldots; q_n = \frac{Q_n}{S_n\left(\pi d_n H_n + \frac{\pi d_n^2}{4}\right)}; \quad q_{\text{mean}} = \frac{q_1 + q_2 + \ldots + q_n}{n}$$

decimal six-member scale, permitting a sufficiently complete evaluation. From the treatment of the data available on the inflows to the wells, the following gradations of specific water content may be adopted (A is a univocal coefficient).

Table 1

	q, m³/hour
1. Very high	10 A
2. High	1 A
3. Sufficiently high	0.1 A
4. Medium	0.01 A
5. Low	0.001 A
6. Very low (wells practically dry)	0.0001 A

The evaluation of the water content of a hydrogeological province as a whole and of its separate horizons naturally requires as many data as possible. The more extensive the data used in the determination of the value of water content of an aquifer, the more objective are the results obtained. Hence it follows that an evaluation of aquifers is intimately related to and practically determined by the extent to which the aquifer has been studied, and may vary in accordance with the amount of newly obtained data.

Thus, the evaluation of the water content of an aquifer should be in each separate case referred to a definite term.

The mode of applying the method suggested by the author is illustrated in what follows on the example of the underground waters of the Ukrainian crystalline massif.

The estimation of the water content of the fissure waters of the pre-Cambrian of the Ukraine has been made according to the state of knowledge attained by 1940—1941.

In order to calculate the specific water output data concerning 41 wells have been used. The factual material is not cited for lack of space, only the results of calculations being given. From Table 2 it will be seen that the majority of the holes of the Ukrainian crystalline massif (45.2 per cent) have a specific water output of an order of 0.01 A m³/hour, where A is a univocal coefficient. A considerable part of the wells (28.6 per cent) have a still lower water output (0.001 A) approaching that of rocks containing little or almost no water at all (q=0.0001 A); finally, a comparatively small part (14.3 per cent) show a sufficiently high specific water output (up to q=0.1 A). From the data obtained, as well as from material available on the productivity of the rest of wells, it may be assumed that in the general case the fissure waters of the pre-Cambrian of the Ukraine are no abundant ones.

Notice should be made also of the characteristic feature in the distribution of wells with a different water content over the territory of the massif.

The most productive wells are located in the area of development of ancient granite-gneisses (Znamenka, Bershan) or in the zone of crushing (Klesovo, Ovruch).

Wells with a medium specific water output predominate in the area of development of the Korostyshev-Zhitomir type of granite, the Teterev-Bug series (Rykhalskaya, N. Volynsk, Polonnoye, Zhitomir, Skochitsche, Chervonnoye, *etc.*), old granite-gneisses (Maidan-Villa, Olevsk, Andrushevka, Skvira, Mironovka,

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where q_1, q_2,... are values of the specific water output determined according to strata; and d, diameter of the well.

In the above general formula the term $\pi d^2/4$ may be omitted in this particular case when q is calculated for data of pumpings from wells, and must be taken into consideration when the calculation is made from the results of pumpings from workings with a large diameter (shafts, test-pits, wells *etc.*).

etc.) or in the zone of crushing (Belokorovichi). Finally, wells with a low specific water output occur in the regions of a deep plunging of the pre-Cambrian (Semigorie, Igren); in comparatively rare cases in areas of development of the Korostyshev-Zhitomir (Kirovsk) type of coarse-grained granite (Fastov-Kirovograd), and more frequently in the regions where young intrusions of rapakivi are present (Boguslav, Vladislavovka, Shpola).

The data available show that charnokites and garnet-cordierite rocks (granites) are distinguished by a low water content, although a number of wells in Berdichev and at other points met in these rocks sufficiently abundant waters. Wells with a low specific water output were found in N. Miropol, Chudnov, Troyanov, Vinnitza, Nemirov.

It would be very interesting to compare the specific water output of the fractured rocks of the Ukrainian crystalline massif with that of the fractured rocks of other regions of the USSR (Table 2).

There are comparatively numerous data available on the Dnieper-Donetz depression and the near-Moscow basin, and only scarce information on the wells of the near-Black Sea depression and the eastern slope of the Urals. Therefore the comparisons given below are rather provisional*.

From the figures tabulated it is evident that the specific water output of the pre-Cambrian rocks of the Ukrainian crystalline massif compares unfavourably with that of the wells in the near-Black Sea depression, Dnieper-Donetz depression and even that of the eastern slope of the Urals, occupying one of the last places (Table 2).

Table 2

Province	$q=10\ A$	$q=1\ A$	$q=0.1\ A$	$q=0.01\ A$	$q=0.001A$	$q=0.0001\ A$
Ukrainian crystalline massif .	—	—	14.3	45.2	28.6	11.9
Near-Black Sea depression (eastern part)	—	57.0	29.0	14.0	—	—
Dnieper-Donetz depression (Cretaceous)	—	11.0	78.0	5.5	5.5	—
Do (Devonian)	—	18.2	72.7	9.1	—	—
Near-Moscow depression . . .	1.3	20.8	61.0	15.6	1.3	—
Eastern slope of the South Urals	—	—	71.4	28.6	—	—

The data of Table 2 show that the fractured rocks of the pre-Cambrian of the Ukraine are the least abundant in water; the limestones of the eastern part of the near-Black Sea depression (Melitopoltschina) are distinguished by a high water content; somewhat less water is contained in the fractured marlaceous chalk formation and the limestones of the Devonian of the Dnieper-Donetz depression, and very high values of water output are observed in the near-Moscow depression.

The fractured rocks of the eastern slope of the Urals (siliceous shales, jaspers, porphyrites, different tuffs, diabases, *etc.*) contain more water than the crystalline pre-Cambrian rocks of the Ukraine.

In fact, enormous discharges of wells are observed in the near-Black Sea area (Chaplinsk, Skadovsk, N. Troitzk regions). These wells secure the irrigation of vast cotton fields and of other crops requiring watering.

The waters of the marlaceous-chalk formation and limestones of the Devonian of the Dnieper-Donetz depression are in many cases the sources of water supply of large populated localities and of industrial enterprises (Kharkov, the industrial undertakings of the marginal parts of the Donetz basin, the industrial enterprises of the towns of Bryansk, Orel, *etc.*).

* For the Dnieper-Donetz and the near-Black Sea depression the specific outputs of wells have been computed from data for 1940—41([2], [3]), for the near-Moscow basin for 1939—40([1]), and for the Urals, for January 1, 1942.

The big industry and the towns of the Moscow region also use successfully the large inflows of the wells exploiting the waters of the Carboniferous of the near-Moscow basin.

The fissure waters of the Ukrainian pre-Cambrian may only in rare cases be used as a source of water supply for large towns (Berdichev, Zhitomir), and to obtain in this case the amount of water required it is necessary to drill a large number of wells (making large capital investments). It is only the absence of other aquifers more abundant in water at a number of points that makes drilling of wells for the fissure waters of crystalline rocks to be recommended. Consequently, among the hydrogeological provinces of the European part of the USSR the Ukrainian crystalline massif occupies one of the last places; the specific water output of the crystalline rocks ranges within 0.01 A—0.001 A, and the average productivity of the wells is 8—10 m^3/hour.

From all the aforesaid it follows that the proposed method for the evaluation of the water content of the aquifers and the hydrogeological provinces as a whole is based on comparisons of univocal values calculated from experimental data, the results obtained reflecting with sufficient fullness the extent of knowledge concerning the underground waters of the region.

From a comparative evaluation of the water content of aquifers, a sufficiently full and objective idea may be formed of the abundance of the aquifer, which is very important in order to turn to profit the underground waters of the region.

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GEOLOGY

PROSPECTS OF THE COAL DEPOSITS OF AZERBAIDJAN

By R. G. SULTANOV

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Until now no sufficient attention has been given to the study of the coal deposits in Azerbaidjan.

Taking into account the insufficient geological knowledge of Azerbaidjan, work in this line has been considered unadvisable.

Using the method of correlation, we shall dwell here at some length on the geological data for Georgia and Central Asia, where the following deposits are known and worked: Tkvibuli, Tkvarcheli, Bzybskoye, Maganskoye, Chalskoye, Gelatskoye, *etc.*, as well as the Ghissar and Zeravshan coal-bearing regions, *etc.*

All of these beds are confined to Jurassic deposits which, like the other Mesozoic deposits of the Caucasus and Central Asia, until now have not been sufficiently studied either stratigraphically or facially, save for a few deposits.

In Georgia, the coal-bearing deposits are in the main represented by the following section (in descending order) (1):

1. A clastic series consisting of grey clayey sandstones interbedded with conglomerates and passing lower down into compact greenish conglomerates.

2. A coal-bearing series of Jurassic rocks represented by alternating clayey tufaceous sandstones, clay and coal shales, enclosing coal beds. The thickness of the coal-bearing series is not persistent, neither are its sequence and alternation of the composing rocks along the vertical and the horizontal, even within the same deposit; it has also different coefficients of coal content. The thickness of this series varies within 100—300 m, reaching its minimum in the area of development of the shale facies, and considerably increasing in the area of the sandstone facies.

3. A clastic tufaceous series.

In the coal-bearing series the presence of 31 species of plants has been recorded, such as ferns, cycads, gingkos and conifers.

The coal deposits of Georgia refer to the allochthonous type (1), the conditions of every individual basin having repeatedly changed at the period of accumulation of the coal-bearing sediments. The composition of the coal beds was therefore rapidly changing in the process of their accumulation, even within a close distance from one another.

The occurrence of remains of ammonites among clastic rocks of the lower series points to the marine coastal nature of the conglomerates.

It has been established that in Georgia, at the period of formation of the coal-bearing deposits, there existed a number of isolated basins where an accumulation of vegetable remains was proceeding, with subsequent transformation processes of coal formation.

In the accumulation of coal beds in the coal deposits of Central Asia (5), ferns, equisetles, *Cycadophyta*, gingkos and conifers also took part. The coal deposits are of a locally allochthonous nature, the bulk of the vegetable material having accumulated in an autochthonous way. Here the Jurassic coal deposits have a very limited number of outcrops as a result of their being overlain with Cretaceous and Tertiary deposits.

The search work was started here because of the presence of favourable facial relations and structures, irrespective of whether there were direct outcrops on the surface of coals or on the surface of Jurassic deposits which have a discontinuous occurrence.

Without dwelling in more detail upon the coals and the Jurassic deposits of Georgia and Central Asia, let us consider the deposits of the Jurassic in the Little Caucasus of Azerbaidjan.

The Jurassic deposits in the region of Dashkesan (2) are lithologically represented by the following section (in ascending order):

1. A series of quartz porphyries exposed as small windows on the Gandiachai, overlain with conglomerates.

2. A series of tufaceous rocks, porphyrites and their tuffs, sandstones with ammonites and belemnites of Middle Jurassic age, about 640 m thick, covered with a series of grey limestones of Upper Jurassic age. Effusions (2) of porphyrites of the Middle Jurassic were of a subaqueous nature in a shallow-water basin and near the coast, which is evidenced by the remains of silicified trees, probably carried in here from the near-lying land.

At a number of points in the Little Caucasus Abich, Florensky, Barsanov, Barklay and Kerselidze found an abundant flora, which, as determined by Krishtofovich, Prinada and Bogachev, refers to *Cycadophyta*, ferns, conifers and gingkos. In the same region an abundant fauna of ammonites has been discovered, determined by Bogachev, which according to their living conditions belong to the coastal zone.

From their fauna and flora, these deposits of the Little Caucasus may well be correlated with the coal-bearing series of Georgia (3) and Central Asia (5).

Literary data on the Jurassic deposits of the Little Caucasus are sufficient for a preliminary correlation and for inferring that the climatic conditions at this period of the Jurassic were identical in Azerbaidjan, Georgia and Central Asia.

In other words, throughout the belt of Central Asia—Azerbaidjan—Georgia, the bulk of the remains of the Jurassic flora consists of the same plants.

Along with the data furnished by the fauna and flora in this area of the Little Caucasus, there are a number of coal occurrences, which are of interest according to archive and recent data.

These are as follows:

1. In the Khandar region, near the Bayan village within the wintering place of Chinary-Gomer; the Zaglik deposit in the Khenani-Kakh locality.

2. In the Shamkhor region, the Irmashla outcrop of coals in the Gay-Bulakh locality. The outcrop has the form of a pocket about 2.15 m long and 0.3 m thick. The Kabakhtapinsky outcrop of coal on the Koshkar-chai river.

3. In Kurdistan, in the Kochtash locality and near the Shapiksit village.

4. In the Djabrail region, the Khudafera outcrop of coal, within 400 steps from the bridge across the Araks river, near a small hill at the boundary with Iran. According to personal information by Bogachev, the outcrop of coal has an apparent thickness of 0.8—1 m.

5. In the Kedabek region, near the Novosaratovka and Novoivanovka villages. In the same region, according to the communications of K. Khydyr-zade, east of the Miskinly village, 0.5—1 km beyond the Sary-yal pass, in the bed of the gorge there are exposures of coal beds. At the same place there are also outcrops of bluish unctuous clays. At the surface there are blocks of coal, 1—1.5 m^3 in volume, which have rolled down. The coals are dull or lustrous with a low specific gravity.

In the same region, according to information by M. Abdalov, agronomist, in the Gay-Deya locality, there is an outcrop of coal.

6. In 1940 in the Upland (Nagorny) Karabakh, near the Chardakhly village, a deposit of coal was opened up, which, according to the communication of Sultanov and Suleimanov, was confined to the Middle Jurassic and showed the following section (in descending order):

1) a tufaceous series, tuff-sandstones, conglomerates, tuff-breccias, thickness 800—900 m;

2) andesite-basalts, thickness 200—300 m;

3) sandstones, in the upper portion partly tufaceous, thickness 60—70 m;

4) alternating clayey shales, sandstones and clays. To the latter series coal beds are confined.

In the small workings driven the thickness of the coal beds ranged from some centimeters to 0.8 m. The coal beds are markedly different also with regard to the enclosing rocks. All the deposits are broken by transverse fractures and in places heavily crumpled.

The coals are black-coloured, compact, massive, with a lustrous surface and without well-marked admixtures.

Thus, considering the identity of physical and geographical conditions throughout the belt of Central Asia—Georgia, which geologically seem to continue each other, similar conditions of accumulation and sedimentation must have existed at the period of coal accumulation of the Middle Jurassic. In this respect it should be taken into consideration that the «base level of wave action» reaches to depth of 200 m. Hence the conditions of accumulation of the coal material in the form of lagoons outside the sphere of the action of the waves.

In the course of further work these data require that the direction of the removal of the material along the dip or the strike should be ascertained.

In the Little Caucasus the principal part of the exposures of the Middle Jurassic is represented by coastal deposits in the zone of the «base level of wave action»; this may be taken to mean that a closed relief existed in the places of coal-accumulation.

These data are being supported by considerations of a purely geological nature. The region of exposures of Georgian coal-bearing deposits is confined to the Rion plate (4).

In the centre of the plate there are vast outcrops of Jurassic coal-bearing deposits, forming slightly compressed folds.

Of frequent occurrence are vast complicated synclines and dome-shaped anticlines with wide arches, complicated on the limbs by secondary folding and dislocation.

The process of erosion affects here directly Jurassic coal-bearing rocks, owing to which exposures of coal beds are frequent.

The Azerbaidjan plate(4) is characterized by a somewhat different folding. Here the principal features of the folding consist of overthrusts of the northern part over the underthrust southern portion. At places, wide anticlines and synclines with almost horizontally lying rocks are formed. As a result, erosion was able to expose the Jurassic deposits only at some places, while its chief destructive activity affected the underthrust Cretaceous deposits exposed by it.

On the Rion plate open structures have been produced; in Azerbaidjan, closed ones.

It should be noted that the structure of the southern part of Nagorny Karabakh approaches that of the Rion plate the most closely.

In comparing the data on the coal-bearing deposits of Central Asia with those on the Jurassic deposits of Transcaucasia, many features are found to be in common with the Little Caucasus of Azerbaidjan, as regards their physical geography, conditions of accumulation, facies, structure, *etc.* This justifies the assumption that in the Little Caucasus of Azerbaidjan there also exist coal-bearing deposits, covered by the overlying Cretaceous and Upper Jurassic deposits.

In confining ourselves to these descriptions of the Jurassic deposits of Georgia and Central Asia, as compared to those of the Little Caucasus of Azerbaidjan, it should be noted that at present there are data available that bear evidence of the presence in Azerbaidjan, too, of coal deposits of the buried type with closed structures.

In order to settle the question as to the presence of coal beds in the Jurassic deposits of Azerbaidjan, a study should be made of the extensive belt of outcrops of Middle Jurassic rocks of the Little Caucasus, of the facial relationships along the dip and the strike, of the features indicating the presence of coal, and of the suitable geological structures.

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GENETICS

PRODUCTION OF AMPHIDIPLOIDS OF *TRITICUM POLONICUM* × × *TRITICUM TIMOPHEEVI*

By A. R. ZHEBRAK, Member of the Academy of Sciences of the White Russian SSR

In 1939 *T. Timopheevi* was crossed with *T. polonicum* and hybrid seeds resulted from this cross. For the purpose of crossing with *T. Timopheevi*, a variety of *T. polonicum* with white awn, white grains and yellow-green, glaucous leaves was selected. Besides these plants of the pure species we had also second generation hybrids *T. polonicum* × *T. durum melanopus* 069 which were referred by us to the type of *T. polonicum*. These were also used by us for crossing with *T. Timopheevi*. The hybrid grains produced during the 1940 growing period were treated with colchicine solution at the stage of grains and seedlings. In our experiments No. 24 and 87 we treated the seedlings, while in experiments 57 and 101 first dry seeds were treated and later the treatment was repeated on seedlings. For the purpose of recording taxonomic characters we recorded only characters of amphidiploid plants and seeds obtained from the crosses of *T. Timopheevi* with pure species of *T. polonicum*, and not with hybrid plants (obtained from F_2). The combination of *T. Timopheevi* with the type *T. polonicum* from F_2 was of interest to us as regards the development of some other characters. The scheme of experiments with colchicine treatment is given in the table.

An analysis of the numerical data shows that the number of amphidiploid ears is a rather considerable one.

The leaves, culms and glumes of the F_1 plants *T. polonicum* × *T. Timopheevi*, as well as in any other combination involving *T. Timopheevi*, are pubescent. The variety of *T. polonicum* selected had soft glumes. The glumes of the spikelets in amphidiploid ears were hard and had a long tooth. The length

of the glumes without tooth was 14—16 mm, the length of the tooth was from 2 to 3 cm. The ear was brittle, more brittle than that of the amphidiploid *T. durum* × *T. Timopheevi*.

The ear shown in the figure had 13 productive spikelets. These 13 spikelets had a total of 23 grains. The grain were oblong, narrow, rather plump. The average length of the grain reached 10—12 mm, its width 3—3.5 mm, its height 3—3.5 mm. The germ made up slightly over one quarter of the total length of the grain. The absolute weight was calculated from weighing all the amphidiploid grains of a single plant, which totaled 56 grains. The weight of these 56 grains was 3.21 g, which would make 57.3 g per 1000 grains. The amphidiploid grains of *T. polonicum* × *T. Timopheevi* were on the average better filled than those of amphidiploids of other combinations.

The absolute weight of 1000 grains, as found from weighing large numbers of grains, proved to be considerably higher in 1942. Thus, for example, in the amphidiploid plant B-57 it attained 80 g; the amphidiploid B-87 showed a weight amounting to 76.5. The mean number of grains per ear amounted in 1942 to 31.2; that per spike was equal to 2.

The F_1 hybrid plants of *T. polonicum* possessing amphidiploid ears were under glasshouse conditions not affected by mildew. Under exactly similar conditions plants of *T. vulgare*, *T. durum* and their hybrids grown side by side to them were severely affected.

Morphologically the amphidiphoids of *T. polonicum* × *T. Timopheevi* can easily be distinguished from the amphidiploids of other interspecific hybrids of wheat by the character of their glumes. At early stages of development up to the formation of ears there is no difference between the amphidiploid plants of *T. polonicum* × *T. Timopheevi* and those of *T. durum* × *T. Timopheevi* as regards the colour of leaves and the character and degree of pubescence.

Combination and number of experiment	Number of hybrid seeds under exper.	Concentration of colchicine solution, %	Duration of treatment, in hours	Number of plants that survived	Number of completе amphidiploids	Number of plants with amphidiploid ears	Total number of ears	Number of amphidiploid ears	Number of seeds in amphidiploid ears	Number of absolutely sterile ears	Number of ears with few seeds	Number of seeds in such ears
Type *T. polonicum* from F_2 × *T. Timopheevi*												
No. 24	34	0.025	48	3	—	2	32	2	13	25	5	8
No. 57	43	0.0125	20	35	—	30	335	112	1747	198	25	53
		0.0125	40									
T. polonicum × *T. Timopheevi*												
No. 87	5	0.025	44	3	—	3	16	11	151	3	2	2
No. 101	5	0.0125	17	2	1	2	8	4	40	1	3	5
		0.025	48									

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CYTOLOGY

BASOPHILY OF PROTOPLASM IN *PROTOZOA* AS CONNECTED WITH THE OCCURRENCE IN THE CELL OF ZYMONUCLEIC ACID

By G. I. ROSKIN and A. S. GINSBURG

(Communicated by L. A. Orbeli, Member of the Academy, 26.VI.1943)

In an overwhelming majority of *Protozoa* the cytoplasm is basophilic. One easily becomes aware of this fact by applying to the investigation of most various *Protozoa* the same colour methods as are commonly used in cytology in order to decide, whether a given element is basophilic of oxiphilic. This peculiarity of the protozoic protoplasm becomes still more remarkable (we understand here the relative basophily, and not the absolute one) if it be compared to the cells of polycellular animals, in which the basophily of the protoplasm is observed rather as an exception, typical of a relatively restricted group of cells.

Our first attempts to analyse the plasm in *Paramoeciae*(1) revealed in their cytoplasm the existence of two components responsible for the effect of basophily. One of these components is the zymonucleic acid, while the other seems to be some basophilic proteins which dissolve when treated with hot distilled water. This fact made us study various *Protozoa*, both free-living and parasitic, by a method applied in our previous work and based upon investigations by Dubos and MacLeod (3) and Brachet (2).

We have studied the free-living infusoria *Euplotes* sp. and the parasitic *Opalina ranarum*, an amoeba of the type *limax*, the rabbit coccidia *Eimeria stidae*, *Trypanosoma lewisi* and, lastly, *Plasmodium vivax*. The material was fixed with 96° alcohol. Preparations treated with the enzyme ribonuclease, and control preparations treated with distilled water, as well as such as underwent no preliminary treatment whatever, were stained after Dominici, Giemsa, or with methylgreen-pyronine according to Unna.

The investigation of the infusoria *Euplotes* in smears stained after Dominici showed its plasm to be basophilic and to stain pale-blue with toluidin blue; its micro- and macronucleus stain intensely taking on a deep-blue colour. Preliminary treatment with hot distilled water (Fig. 4, *a*) makes the colour somewhat less intense without changing, however, its character. Upon treatment with ribonuclease (Fig. 4, *b*) the basophilic substance of the plasm vanishes entirely, the cytoplasm stains pink with eosin. The micronucleus stains invariably dark-blue, while the macronucleus displays, instead of the dark-blue homogeneous mass it was before, a granular structure. Thus, the effect of ribonuclease is to deprive the macronucleus of one of its components, *viz.* the zymonucleic acid, while chromatin proper is retained by it.

The smears of the parasitic infusoria *Opalina ranarum* were stained with methylgreen-pyronine according to Unna. On preparations subjected to no preliminary treatment or treated with hot distilled water (Fig. 1,*a*) the endoplasm of the infusoria may be seen to contain a large amount of basophilic substance

which stains with pyronine an intensely red colour. This substance is distributed diffusely, forming occasional clots. The nuclei stain a greenish-blue.

After having been treated with ribonuclease (Fig. 1, *b*) the plasm stains only a pale-pink, and the main basophilic substance disappears. This gives us ground to believe that the cytoplasm of *Opalina* is very rich in zymonucleic acid. The treatment with ribonuclease fails to have any effect upon the colour of the nuclei.

The next object of our investigation were amoebae of the type *limax*, taken from a film over hay decoction. The smears were stained after Dominici. The basophilic substance which is diffusely distributed throughout the plasm assumes a pale-blue colour when stained with toluidin. The kariosome of the amoeba stains an intense dark-blue. The treatment of the preparations with hot distilled water does not change the nature of its colour appreciably, while under the

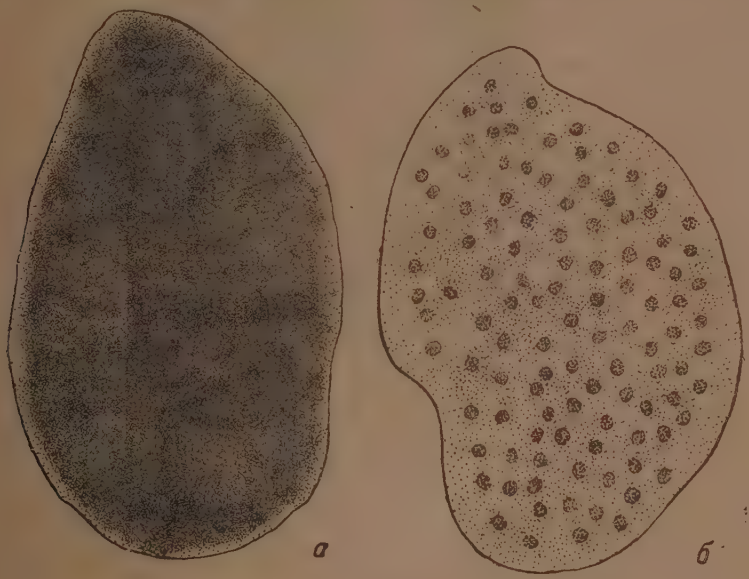

Fig. 1

effect of ribonuclease the basophilic component of the plasm disappears entirely and the plasm stains pink with eosin.

The basophilic substance of the kariosome also vanishes during this procedure. Thus, in amoebae the cytoplasm is also found to be impregnated with a certain amount of diffuse zymonucleic acid; if we are to judge by the effect of staining, considerable quantities of it are concentrated in the kariosome. After the zymonucleic acid has been eliminated, the kariosome stains with eosin, *i. e.* it takes in other protein component.

According to observations on *Trypanosomata* (*T. lewisi*), their plasm seems largely to consist of basophilic proteins. When the smears are stained after Giemsa (Fig. 2, *a*), the whole of the plasm is found to assume a light-blue colour.

In some cases, against a light-blue background separate small areas of oxiphilic pink plasm are observable (we do not touch here upon the question as to the rôle of fixatives in the separation of plasmic fractions that may take place here). Upon treatment of the smears with hot distilled water the basophily of the plasm becomes somewhat less intense, while the rest of the elements (nucleus, blepharoplast) do not change their colour. When treated with ribonuclease (Fig. 2, *b*) the basophilic component of the plasm disappears, and the *Trypanosomata* stain a very pale pink. The blepharoplast remains unaltered. As to the nucleus, one has the idea as if the latter becomes poorer in nuclear substance without losing in the intensity of coloration; now it contains «vacuities».

The possibility is thus not excluded (this is, however, no more than a mere supposition) that the nuclei of *Trypanosomata* contain some elements vanishing as soon as the preparation is treated with ribonuclease, *i. e.* consisting of zymonucleic acid.

Among the representatives of *Sporozoa* we have investigated coccidiae from the liver of rabbits. Liver sections were stained after Dominici and Unna. The

Fig. 2

plasm of the oocysts of *Eimeria stidae* was found to stain intensely with pyronine (when stained after Unna), and to assume a dark violet-blue if stained after Dominici (Fig. 3, *a*). Upon a treatment with ribonuclease, the basophilic substance

Fig. 3

disappears utterly from the protoplasm (Fig. 3, *b*). The cytoplasm of the oocysts is thus found to be extremely rich in zymonucleic acid. Along with a diffuse distribution of the zymonucleic acid throughout the plasm, it is not infrequently observed to accumulate in considerable amounts around the nucleus. Such oocysts are also encountered whose peripheral layer of plasm is richer in basophilic substance than the centre of the cell is. Lastly, in some cases zymonucleic acid may be observed to condensate at the poles of the cell, while the rest of the plasm is but poorly perfused by it. The nucleolus of an oocyst contains a large amount of zymonucleic acid.

The malarial plasmodium (*P. vivax*) could be met on our preparations in the shape of young schizonts and at stages of shizogenesis. Smears prepared with the blood of malarial patients were stained after Giemsa. The plasm of the parasites was found to be basophilic. After a treatment with hot distilled water the character of the colour remained unaltered, but the cytoplasm was found to take on a lighter blue shade. A treatment with ribonuclease had no appreciable effect upon the colour of the nucleus, but the basophilic substance disappeared from the protoplasm entirely. The plasm assumed thereby a pale pink-grey colour.

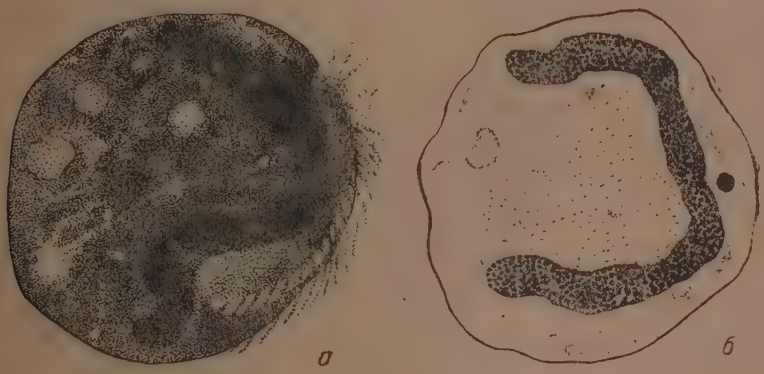

Fig. 4

The observations here described permit of the following conclusions.

The basophily of the plasm in various representatives of *Protozoa*. is largely accounted for by the presence of cytoplasmic nucleic acid. This acid is of common occurrence in *Protozoa*, though its amount, if we are to judge by the effect of the dyes, seems to vary according to the species.

Contrary to the general belief that zymonucleic acid (or a group of compounds closely related with it) is a component specific for the cytoplasm, the presence of this acid can be demonstrated not only in the cytoplasm of *Protozoa*, but also in the macronucleus of infusoriae, in the kariosome of amoebae and in the nucleolus of oocysts of coccidia.

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PLANT PHYSIOLOGY

INTERACTION BETWEEN ROOTS AND SOIL COLLOIDS AS ONE OF THE PROBLEMS OF THE PHYSIOLOGY OF MINERAL NUTRITION OF PLANTS

II. AGE VARIATIONS IN THE FIXING CAPACITY OF THE PLASM

By E. I. RATNER

(Communicated by A. N. Bach, Member of the Academy, 15.VI. 1943)

Unlike the colloids of the soil, those of a vegetable cell are a highly unstable system, reflecting the whole complex of the internal processes that are responsible for the life and development of an organism.

The most general and stable changes in the properties of the biocolloids are those of them which are linked with the age of the organism. At the time being a sufficient amount of evidence has been stored, pointing to that the ageing of a plant or of its separate organs is attended by a reduced activity of the colloidal system.

This reduction may be expressed in various phenomena, in the first line in the degree of hydration of the biocolloids. The critical period of the plant's development in this connexion, as in many others, too, corresponds to the period of flowering([1–6]). In connexion with the problem we are interested in, it should also be emphasized that the ageing of a plant, in particular, its passing to the stage of flowering, is also associated with a considerable abatement of the intensity of respiration of the leaves ([2, 5, 10]). The import of this fact consists in that it is the process of respiration that mainly provides for the supply to the cell of the hydrogen ion, at whose expense proceeds the general exchange of cations between the plant and the external medium; it is also operative in the desorption of the cations absorbed by the plant, from the plasm into the cell sap. The reduction of the respiration intensity of the leaves will not only handicap the work of the conveyer by whose means the migration of the cations from the soil on to the plant is effected; in case the concentration of a given cation in the soil colloids is low, it may create such conditions under which a reverse flow of the cations from the plant into the soil will take place. Thus, age variations are liable to reduce the plant's capacity to fix the cations.

In order to give this question an experimental interpretation, I carried out in 1940 an experiment with barley of the variety Wiener 01163, grown for that purpose in water cultures, on Helriegel's solution. To make the experiment more explicit, the plants were being grown under a profuse supply of nutrient substances (the nutrient solution was replaced periodically by a fresh one). We have already had occasion to learn([7]) that this procedure is beneficial for the development of the process of desorption by the soil of cations from the plant.

At the stages of tillering, flowering and of ripening of seed, the roots of the plants were thoroughly washed from the nutrient solution, after which they

were set in contact with a suspension prepared for this purpose with chernozem saturated to 30 per cent with Ca, and to 70 per cent with Na; they were left to interact for 36 hours, under frequent air blowing. In another experimental set, the plants were immersed with their roots into vessels filled with distilled water, the duration of the test and all other conditions being the same. Analyses made previous to the experiment and upon completion of the latter gave the results tabulated below (Tables 1 and 2).

Table 1

Ca Content in Plants

	Tillering		Flowering		Ripening of seed	
	in mg per vessel*	in % to control	in mg per vessel	in % to control	in mg per vessel	in % to control
Previous to experiment (control)	24.9	100	309.9	100	451.9	100
Upon immersion of roots into distilled water for 36 hours . .	24.5	98	269.8	87	397.3	88
Upon immersion of roots for 36 hours into suspension of soil saturated to 30% with Ca and to 70% with Na	19.6	79	172.8	56	291.0	64

* Each vessel was made to contain 6 plants.

Table 2

K Content in Plants

	Tillering		Flowering		Ripening of seed	
	in mg per vessel	in % to control	in mg per vessel	in % to control	in mg per vessel	in % to control
Previous to experiment (control)	107.3	100	496.1	100	549.8	100
Upon immersion of roots into distilled water for 36 hours . .	104.6	98	439.8	89	490.2	89
Upon immersion of roots for 36 hours into suspension of soil saturated to 30% with Ca and to 70% with Na	93.2	87	321.4	65	422.3	76

As may be seen from Tables 1 and 2, the experimental data go fully to confirm the assumption exposed above to the effect that the fixing ability of the plant with respect to the cations undergoes reduction, according as the plant grows older. This statement is illustrated by the behaviour of the plants as soon as these are transferred to water, for older plants lose thereby an appreciable portion of the cations they contained, while the younger ones fail to do so. This difference becomes still more salient, however, when the roots are made to interact with soil suspension.

Here the peculiar rôle played by the period of flowering becomes especially illustrative ([2],[5],[6]).

Tables 3 and 4 illustrate the variations in the ratio of calcium and potassium in the roots and in the aboveground organs of the very same plants, and some other regularities.

The first to strike the eye in Tables 3 and 4 is the clearly outspoken gradient of the fixing ability of the plant, which is found to increase in the direction from ears to roots. This is typical of any age, both for Ca and K, being more conspicuous in the former. With respect to Ca, an analogous clearly expressed gradient has

Table 3

Content of Ca in Separate Plant Organs, in per cent *

	Tillering		Flowering			Ripening of seed		
	Roots	Above-ground organs	Roots	Stalks and leaves	Ears	Roots	Stalks and leaves	Ears
Previous to experiment (control) . .	1.902	1.084	3.874	1.241	0.270	4.018	1.796	0.289
	100	100	100	100	100	100	100	100
Upon immersion into distilled water for 36 hours	1.714	1.092	2.532	1.129	0.256	2.382	1.729	0.264
	90.1	100.1	65.4	91.0	94.8	59.3	96.2	91.4
Upon immersion for 36 hours into suspension of soil saturated to 30% with Ca and to 70% with Na . .	1.087	0.953	1.354	0.723	0.198	0.840	1.325	0.238
	57.1	87.9	34.9	58.3	73.3	20.9	73.8	82.3

* The numerator gives the absolute magnitudes in grams per 100 g of absolutely dry matter; the denominator, relative ones, the data of the control being put at 100.

Table 4

Content of K in Separate Plant Organs, in per cent

	Tillering		Flowering			Ripening of seed		
	Roots	Above-ground organs	Roots	Stalks and leaves	Ears	Roots	Stalks and leaves	Ears
Previous to experiment (control) .	4.894	5.483	0.685	1.868	1.314	1.387	1.661	1.123
	100	100	100	100	100	100	100	100
Upon immersion of roots into distilled water for 36 hours	4.650	5.377	0.443	1.596	1.287	0.982	1.407	1.085
	95.0	98.1	64.7	85.4	98.0	70.8	84.7	96.6
Upon immersion of roots into soil suspension, saturated to 30% with Ca and to 70% with Na	4.095	4.802	0.326	1.063	1.114	0.663	1.185	0.975
	83.7	87.6	47.6	56.9	84.8	47.8	71.4	86.8

also been observed in experiments involving kenaph, which were carried out under conditions similar to those with barley.

Coming back once more to our data on barley, it should be pointed out, that contrary to the generally adopted belief as to the poor mobility of calcium in the plant, when putting the roots into interaction with the soil colloids, there is observed a well expressed process of desorption of this cation by the soil from the plant, the entire plant from its roots up to the ears being involved in the process.

Now we have to answer two questions: 1) in how far the facts here recorded may be interpreted as phenomena of cation exchange between the plant and the soil colloids; 2) to what extent we are right in assuming the existence of a relation between the drop of the fixing capacity of the plasm, on the one hand, and the decrease in the intensity of respiration of the cells, according as the plant and its individual organs grow older, on the other hand.

An answer to both questions is given in Table 5 which contains data concerning barley plants from the experimental set described above, as analysed not only for their content of Ca and K, but for Mg and Na as well, *i. e.* of all those cations that might have participated in the exchange between the roots of the plant and the soil cations.

Table 5 bears out with particular clearness the fact that within the 36-hour interaction of the roots with the soil colloids, two clearly opposite cation currents make their way through the plant; one of them, made up by ions of Ca, K and

Mg, travels from the plant into the soil, while another, consisting of Na ions, is directed from the soil to the plant. From the difference between the sum total of mgs equivalent of Ca, Mg and K eliminated from the plants and the number of mgs equivalent of Na uptake, we gain an idea as to the part played by the hydrogen ion produced by the cell in the phenomena of the exchange taking place between the plant and the soil colloids.

As may be seen from Table 5, in plants at the phase of tillering the amount of sodium taken up considerably exceeds the total of Ca, K and Mg ions eliminated; with aboveground organs the latter is thrice as large as the former. In other words, for those young plants whose leaves display a highly intense respiration process, the cation exchange with the soil colloids proceeds in the first line at the expense of the hydrogen produced in the cell in the process of respiration.

The picture is quite different with plants at the stage of flowering. Here the number of mgs equivalent of Ca, K and Mg evolved by the vegetative organs is almost equal to that of Na uptaken by them; in roots it is even somewhat smaller. Thus, in these organs the cation exchange went on without participation of hydrogen.

And it is only in the case of reproductive organs that are known for their high breathing intensity at the stage of flowering, that the amount of sodium uptake is found twice to exceed that of the Ca, K and Mg ions evolved. Hence the part played by the hydrogen

Table 5

Cation Composition in Plants Previous to and upon Interaction of Roots with Soil Suspension
(in mg-equiv. per 100 g of dry matter)

	Roots					Stalks and leaves					Ears				
	Ca	K	Mg	Total Ca+K+Mg	Na	Ca	K	Mg	Total Ca+K+Mg	Na	Ca	K	Mg	Total Ca+K+Mg	Na
Tillering															
Previous to interaction with soil suspension	95.1	125.5	10.9	231.5	7.6	54.2	140.6	11.5	206.3	9.8	—	—	—	—	—
Upon interaction with soil suspension	54.3	105.0	7.8	167.1	92.2	47.6	123.1	7.2	177.9	85.4	—	—	—	—	—
Difference	—40.8	—20.5	— 3.1	— 64.4	+84.6	— 6.6	—17.5	— 4.3	+ 28.4	+75.6	—	—	—	—	—
Flowering															
Previous to interaction with soil suspension	193.7	17.6	19.1	230.4	7.4	62.0	47.9	9.1	119.0	5.4	13.5	33.7	2.7	49.9	2.0
Upon interaction with soil suspension	67.7	8.4	14.0	90.1	127.4	36.1	27.3	5.8	69.2	56.1	9.9	28.6	1.4	39.9	23.3
Difference	—126.0	— 9.2	— 5.1	—140.3	+120.0	—25.9	—20.6	— 3.3	— 49.8	+50.7	— 3.6	— 5.1	— 1.3	—10.0	+21.3

ions in the exchange is as conspicuous here as in the leaves of young plants.

From the existence in a plant made to interact with the soil colloids of two opposite cation currents and the interpretation we give to this phenomenon, we are led to assume that the movement of the cations is proceeding along the same living tissue, most probably along the phloem.

This motion may be imagined to take place in the form of a surface migration of ions ([8]), stimulated by a different degree of saturation of the soil colloids with a given cation and by the composition of the other cations ([9]) by which it is attended, on the one hand, and by the state of the colloidal system of the vegetable cell, on the other.

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PHYSIOLOGY

TREATMENT OF HAEMOSPORIDIOSES OF HORSE WITH NEW HOME-MADE PREPARATIONS, NOVOPLASMINE LP_4 AND SPOROPLASMINE LP_3*

By P. I. POPOV and M. W. DEGTIAREV

(Communicated by A. E. Arbusov, Member of the Academy, 13.X.1943)

The deficiency of specific remedies usually applied in the case of haemosporidioses of the horse induced the Kazan Scientific Research Veterinary Institute to organize in 1943 an expedition to the Bilarsky and Novo-Sheshminsky districts of the Tatarian Autonomous Republic, in order to test the therapeutical efficiency of preparations newly synthesized by M. P. Gerchuk (1) at the Laboratory of heterocyclic compounds of the Academy of Sciences of the USSR.

The staff of the expedition, consisting of M. W. Degtiarev, P. I. Popov and N. Helfman, were treating within May and June 66 horses affected with haemosporidioses; 62 of the horses recovered, 4 perished, the ratio of recovery thus amounting to 94 per cent. The animals were kept under regular clinical inspection, their pulse and temperature taken, arterial blood pressure measured by the method of oscillometry, ratic of haemoglobin, rate of settling of erythrocytes were determined. The quantitative and qualitative composition of red blood corpuscles was determined and a haemogram taken. Smears of peripheral blood were examined under the microscope for the presence of haemoparasites and for the kind of the reaction to the parasites.

Each animal was repeatedly examined, previous to the administration of the remedy and on the second or third day after it. In the subsequent the animal was visited at regular time intervals until its recovery, which made it possible to form a judgment as to the course of restoration of the diseased organism.

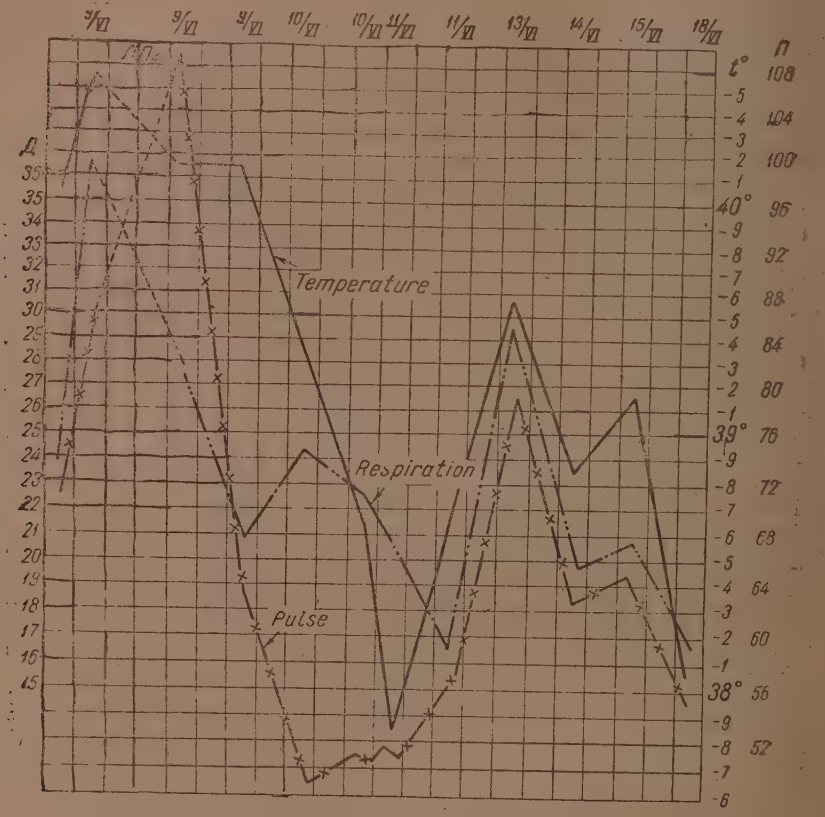
In most cases it was novoplasmine that had to be applied. This is a crystalline substance soluble in hot water; it was administered subcutaneously in an 1 per cent solution, at a dose of 0.0001—0.0002 per 1 kg of live weight. Much less frequently it was sporoplasmine, a white substance, readily soluble in water of room temperature. This was introduced either intraveinously (in two cases), or subcutaneously (in seven cases), as a 5 or 10 per cent solution, at a dose of 0.003 per 1 kg of live weight. In the latter case the procedure was invariably accompanied by an inflammation of the adipose tissue with the formation of an abscess.

As a result of our clinical tests, the new preparations have been found to be highly efficient remedies against the pyroplasmosis and nuttaliosis in horse. Within a few hours upon the injection the general state of the diseased horse changes appreciably: the oppression disappears, the animal becomes active and reactant again. The temperature often falls to the norm towards the second

* Report held at a conference of the Scientific Council of the Kazan Scientific Research Veterinary Institute, on August, 9, 1943.

day, the appetite is soon reestablished, *etc.* The respiration becomes quiet, dyspnoe and rattle disappear; discharge from the nose becomes less abundant.

Soon after the injection of the preparation the pulse grows faster, but within 1.5 or 2 hours it gradually slows down to the norm (see figure). The arterial blood pressure rises (the maximum increases by 20—30 mm Hg of the oscillometer, the minimum, by 30—40 mm Hg). It is the minimum that shows the most conspicuous rise. In the case of haemosporidioses of the horse it is much below the norm (by 20—15 mm Hg), as a rule.



Clinical status of the horse «Mamanka» (piroplasmosis) prior to and upon application of novoplasmine

In the course of the treatment the blood pressure may occasionally become lower (as in the horse «Batrak»). This should be taken to mean that the blood pressure reveals some changes in the trend of the disease that suggest an unfavourable prognostication. The horse «Batrak» perished before long, in fact (nuttaliosis plus infectious anaemia).

The phenomena of conjunctivitis are attenuated, oedema of the lids disappears, *etc.*

As a rule, the new preparations are diuretic, causing sometimes repeated miction.

Under the action of the new preparations the haemosporidia vanish from the blood within a short period of time. Novoplasmine is endowed with an especially high toxicity for the parasites. Thus, the blood of the horse «Mamanka» was

found to contain on June 8 parasites *Piroplasma caballi* at a number of 180 to per 100 fields of vision. On June 9 they were still more numerous, *viz.* 330 : 100. Novoplasmine has been urgently administered. On June 10, along with a considerably improved general condition of the horse, drop of temperature *etc.*, the blood was found to contain *P. caballi* 10 : 100. After a few days (14.VI) the horse recovered and its blood showed no parasites any more. The pathological process has thus been interrupted, which almost invariably occurred in the case the new preparations were applied*.

The effect of the preparation is to cause degeneration and decay of the parasites. Under the microscope a most curious picture of their destruction may be observed.

The treatment stops the decrease in the haemoglobin ratio, and the latter begins gradually to increase. The number of erythrocytes also increases. The haemograph of the white blood, which during the disease displays a sharp displacement either to the right, or to the left-hand side, is also brought to the normal state by the treatment. The reaction of precipitation of erythrocytes, which proceeds at a high rate in the diseased horses, is slowed down.

At rare occasions the treatment takes a longer time (combined invasion, complications); a repeated injection of the preparation is then necessary. The four horses that perished have received it too late, two of them when already at the stage of prostration.

It should be kept in mind that the application of the remedies here described may occasionally be attended with an unusually strong effect: there may occur excitation of breathing, muscular quiver, abatement, the animal may tend to lying down, trampling, *etc.* But these are rather exceptional phenomena, observed in the case of a rapid intraveinal injection of sporoplasmine, or in the case of an exaggerate dose of the preparations applied. Such a reaction which may vary as to its intensity, lasts for some 15—20 minutes, after which the horse becomes quiet again.

From what has been said it should be inferred that, owing to its excellent qualities (water solubility, strong effect in very small doses when applied subcutaneously, high therapeutic efficiency), novoplasmine may claim an important place among the means applied in the case of horse haemosporidioses.

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* The records of our expedition contain a characteristic of the status of each horse, in accordance to every point of inspection.

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ECOLOGY

RELATIONS BETWEEN *EURYGASTER INTEGRICEPS* AND *PHASIIDAE* FLIES PARASITIC OF THE BUG, AND *PHASIIDAE* AS A MEANS FOR CONTROLLING IT

By D. M. FEDOTOV

(Communicated by A. A. Borissiak, Member of the Academy, 14. III. 1943)

The investigation of *Phasiidae* flies, parasitic of *Eurygaster integriceps* aims at bearing out the relations between the host and its parasite, and at clearing up the possible application of the latter as a means for controlling the pest in question. The work was carried on at the Kashka-Darja district of the Uzbek SSR (Expedition of the Academy of Sciences) within 1941—1942. Data (1) reported as to the high death ratio among hibernating bugs in the Krasnodar region put down to their infestation with *Phasiidae*, as well as observations made by the staff of this expedition during 1941 on *Phasiidae* larvae upon crops at the same localities, made us believe this work to be urgently needed. The biology of *Phasiidae* has been studied by several authors [Wassiljew (2) and some others]. Under conditions prevailing in Central Russian Asia the biology of these flies shows some new specific features, ignoring which we cannot use them as a means for the controlling of *Eurygaster integriceps*.

Two forms of *Phasiidae* parasitic of *Eurygaster integriceps* have been found to inhabit the Kashka-Darja district, *viz.*: *Helomyia lateralis* Meig. and *Alophora* (*Phorantha*) *subcoleoptrata* L. (determined by B. B. Rohdendorf). The former has been found to be parasitic of *E. integriceps* by Wassiljew; with respect to the latter, the present author has been the first to do it.

In 1942 the infestation of adult bugs with *Phasiidae* larvae (the figures refer to both species) was 11.8 per cent, *viz.* out of the 721 bugs dissected by us 85 were found to be infested. Out of this number more than 18 per cent fall to the share of bugs feeding upon crops or wild vegetation; what remains falls to the share of hibernating bugs. No parasites have been detected in any of the 123 dissected bugs of the new generation, taken in June from crops in the vicinity of the town Shakhrisiabz. Out of the 48 aestivating bugs dissected by us only 2 have been found infested, which makes scarcely more than 4 per cent. Thus, the activity of *Phasiidae* is at its maximum during the propagation period of *E. integriceps*.

Out of the 85 bugs infested, 69 specimens have been found to contain one larva each, while the rest contained two, three and even four. Some of the bugs were found to contain dead larvae which had evidently been victims of the injurious influence of the host. Similar data have been reported for other parasitic flies by Rohdendorf (3). In such cases the caudal end of the larva is usually located in the funnel formed in the host's tracheal tissue, while the cephalic end is directed backwards.

Even when bearing an adult larva of the parasite, the bug cannot possibly be distinguished from a healthy one by its external appearance. Feeding upon the

haemolymph of its host, the larva does not affect any of its internal organs. The bug's nutrition proceeds well, its intestine is functioning normally, although occasionally it is strongly pressed together and shifted towards the upper body wall by a large larva. The excretory system, probably because of the intoxication of the bug's organism, is working more intensely. Much excretum is therefore being accumulated in the main truncs of the Malpighian tubes and in the rectum. The amount of the fat body decreases. The presence of the parasite does not always, however, result in a sharp reduction of its quantity. Alongside of infested bugs that were almost utterly lacking any fat body, such were also encountered as contained it in large amounts. This depends on the stage at which a given bug has been infested by the parasite. Histological investigations will certainly reveal alterations in the internal organs of a bug infested with *Phasiidae*; but it is only in the sexual apparatus of the host, and in that of the female in particular, the external changes are clearly observable.

Infested females usually show an underdeveloped ovary which looks suppressed and exhausted. In such a state they were found by us in 51 females; in 5 the egg-tubules were found to contain single green eggs, and only in 2 the ovary was normally developed and contained ripe eggs. Depression and degeneration of the ovary is a phenomenon observed not only in bugs invaded by an adult parasitic larva. Even a young one, no more than 1 mm long, is liable to stop the development and functioning of the ovary in an infested bug. Feeding on the nutrient substances of the host's haemolymph, the parasite seems indirectly to affect the development of its ovary; on the other hand, by excreting toxines it probably impedes the development of the ovary, in order to turn to better profit the haemolymph of the host. As a result, the egg-tubules of the bug become grey-white, flabby, their bead-like structure is scarcely discernible, and their efferent parts strongly swollen. These are usually found to contain near the boundary of the first chamber remains of degenerating eggs in the shape of yellow, flat, rounded corpuscles. The bursae are usually brownish, crumpled, angular. The *receptaculum seminis* is whitish, which points to the fact that the infested females are able of copulation.

Facts of a partial oviposition by an infested female have been reported by some writers. This may take place if the bug-female at the moment of infestation already possesses either a considerably developed, or a ripe ovary. The effect of the parasite, however, is to induce an anomalous development of the ovary: while part of the egg-tubules goes on developing, the rest undergo degeneration without having produced ripe eggs. If an infested female fails to withstand the parasite, it does not propagate and perishes.

The testes of the infested males probably also stop functioning and degenerate, which should be checked up histologically. Their testes, however, do not seem to be badly affected by the parasite.

In those cases when an adult larva escapes the host not through its anal opening, as described by Zwölfer (4), but by expulsing the host's anal segment, the host perishes. It continues creeping about for some time, but there is scarcely any fluid left in its body cavity, its internal organs dry up soon, the exhausted sexual glands adhere to the wall of the body, and the insect perishes.

Females are much more susceptible to infestation with *Phasiidae* than males. Out of the 76 infested bugs taken at random for dissection, there were 61 females (80 per cent) and only 15 males (20 per cent). This may easily be understood since the bug-female offers a more favourable medium for the development of the parasite. The *Phasiidae* belong to those biotic factors which go to reduce the number of females during the period of the active life of *Eurygaster integriceps*.

The relations between the host and the larval stage of the parasite seem to be fairly well balanced, which may be inferred from the fact that the bugs are found to contain dead larvae but on rare occasions.

The peculiarities of the biology of *E. integriceps* are connected with the natural conditions of Central Russian Asia. In particular, the extension of the period of awaking over more than two months, which is due to the high altitude the

bug chooses for hibernation, strongly affects the biology of the *Phasiidae*. The terms of hatching and the period of the development of the generations of *Phasiidae* are therefore also liable to extreme variations. The transfer by the bugs during their spring migration from the mountains and foothills down to the valley of *Phasiidae* larvae, originating from flies inhabiting mountains and foothills, where they use to meet upon crops progeny of valley flies, makes the picture of the *Phasiidae* biology still more intricate.

It was during the second fortnight of March that the first flies made their appearance at the laboratory. Under natural conditions the flies occur on flowers of wild plants, especially on *Ferula*, as well as on various crops. They are distributed all over the Kashka-Darja region, mostly in the mountainous zone, occurring in huge numbers at such places poorly inhabited by *Eurygaster integriceps*; probably, they are parasitic there of some other bugs. Aside from *Helomyia lateralis* and *Alophora subcoleoptrata*, that were hatched from *Eurygaster* bugs, Arnoldi found one species more, the *Rhodogyne clavatum* Rohdendorf; this should be taken to mean that further founds are likely to be expected.

During the active life of the bugs the flies do naturally keep mostly at the crops. In spring at the time when the bugs awake, the *Phasiidae* larvae may be observed to leave the bugs in considerable numbers, and to transform into puparia. The period of development of a puparium is complete within 9—16 days, according to the altitude of the zone and to season.

The considerable rate of infestation of *E. integriceps* with *Phasiidae* observed in the summer 1942 upon the crops of the Kashka-Darja region (somewhat above 18 per cent), the extremely high rate of infestation in the Krasnodar region (80—85 per cent), the wide distribution of the fly in regions, inhabited by *E. integriceps*, along with the data here exposed, point to the necessity and possibility of using these flies as a means to mitigate the pest. Their biology is teaching us how to do it.

In spring, when collecting the bugs that are awaking at their hibernation or on fields, prior to destroying the pest, it is necessary to organize catching of *Phasiidae* flies. The bugs collected must be spread in a layer several cm thick in boxes with perforated bottoms. The openings should not exceed 7.5×4.5 mm. By shaking the box, adult larvae of *Phasiidae*, which escape from the bugs, and puparia that have formed within this time, can be picked out and collected. The puparia collected are to be buried in a shadowy place near the field, covering them with a thin layer of soil.

It is besides necessary, when collecting large numbers of *Eurygaster* bugs for biological laboratories in autumn and winter, and when distributing the bugs in boxes, to collect puparia of the *Phasiidae* flies. Until a method for breeding these flies under laboratory conditions is developed [some suggestions to this effect have already been made ([5])], the puparia collected are to be kept in small boxes filled with sand under natural conditions, in order to transfer them to the field at the approach of the spring.

In order to be efficient for agricultural practice, this method of applying *Phasiidae* as a means for the control of *Eurygaster integriceps*, which is suggested by the biology of the fly, stands in need for further technical improvement.

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